Final Revision

power

→ Electric ⟹ Midterm Revision ملزمة

+ capacitance ملزمة

→ Laplace + poisson → هذه الملزمة

→ Boundary Conditions → هذه الملزمة

Magnetic

- → Biot-Savart
- → Ampere
- → Coaxial Cables
- → Inductance → هذه الملزمه
- → Magnetic circuits → هذه الملزمة

الاثبات في ملزمة الشرح هام

Poisson's & laplace's equations

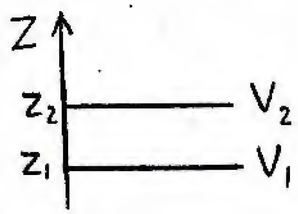
$$\nabla^2 V = \frac{-\rho_v}{\epsilon} \qquad \text{Poisson eq.}$$

$$\nabla^2 V = 0 \qquad \text{laplace eq.}$$

In This part, V is Function of one Variable only.

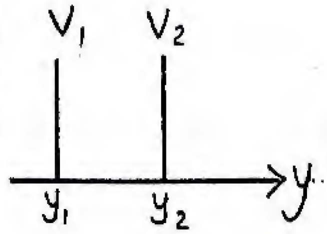
1) determine The Variable
2) Integrate & use The boundary Conditions
3) $\bar{E} = -\nabla V$
4) $\bar{D} = \epsilon \bar{E}$; $\epsilon = \epsilon_0 \epsilon_r$
5) $\rho_s = \pm |D|_{\text{conductor surface}}$, $Q = \iint \rho_s \, ds$
6) $C = \dfrac{Q_{+ve}}{\text{Potential difference between Two conductors}}$

Different Conductor Configurations



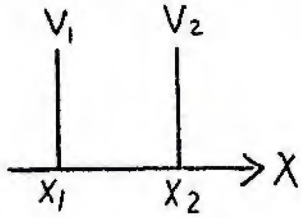
- $V = f(z)$
- $\dfrac{d^2V}{dz^2} = 0$

$V = Az + B$



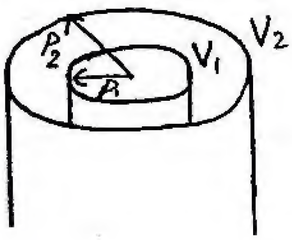
- $V = f(y)$
- $\dfrac{d^2V}{dy^2} = 0$

$V = Ay + B$



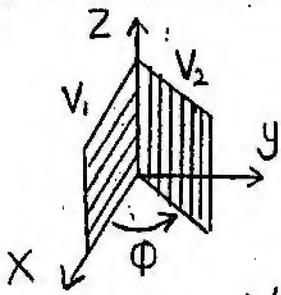
- $V = f(x)$
- $\dfrac{d^2V}{dx^2} = 0$

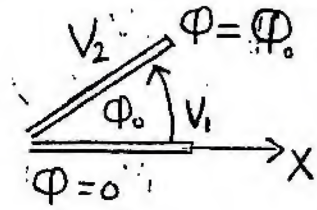
$V = Ax + B$



- $V = f(\rho)$ [cylindrical]
- $\dfrac{1}{\rho}\dfrac{d}{d\rho}\left(\rho\dfrac{dV}{d\rho}\right) = 0$

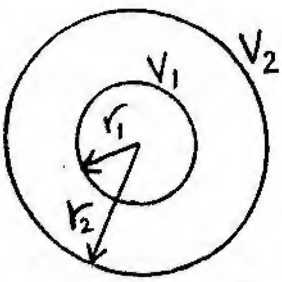
$V = A\ln(\rho) + B$





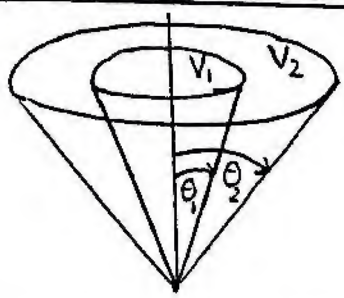
- $V = f(\phi)$ [cylindrical]
- $\dfrac{1}{\rho^2}\dfrac{d^2V}{d\phi^2} = 0$

$V = A\phi + B$



- $V = f(r)$ [spherical]
- $\dfrac{1}{r^2}\dfrac{d}{dr}\left(r^2\dfrac{dV}{dr}\right) = 0$

$V = \dfrac{-A}{r} + B$



- $V = f(\theta)$ [spherical]
- $\dfrac{1}{r^2\sin\theta}\dfrac{d}{d\theta}\left(\sin\theta\dfrac{dV}{d\theta}\right) = 0$

$V = A\ln\left(\tan\dfrac{\theta}{2}\right) + B$

Ex: A spherical capacitor is formed by two concentric spheres of radii $a = 3$ cm & $b = 5$ cm, when the region between the spheres is filled with a dielectric $\epsilon = 6\epsilon_0$,

$V = 100$ Volt @ $r = 3$ cm, $V = 0$ @ $r = 5$ cm

a) The potential field

b) The electric field intensity

c) The capacitance of the system

$$\nabla^2 V = 0 \qquad , \qquad V = f(r)$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = 0 \Rightarrow \frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = 0$$

$$\xrightarrow{\int} r^2 \frac{\partial V}{\partial r} = A \Rightarrow \frac{\partial V}{\partial r} = \frac{A}{r^2}$$

$$\xrightarrow{\int} V = \frac{-A}{r} + B$$

$$V(0.03) = \frac{-A}{0.03} + B = 100$$

$$V(0.05) = \frac{-A}{0.05} + B = 0$$

$\therefore A = 0.6 \qquad B = -20$

a) $V = \frac{0.6}{r} - 20$

b) $\bar{E} = -\nabla V = \frac{-\partial V}{\partial r}\hat{a}_r = \frac{0.6}{r^2}\hat{a}_r$

c) $\bar{D} = \epsilon \bar{E} = \frac{3.6\epsilon_0}{r^2}\hat{a}_r$

$\rho_{s_{+ve}} = + |D|_{r=0.03} = 3.54 * 10^{-8}$ c/m²

$C = \frac{\rho_{s_{+ve}} * 2\pi(0.03)L}{100 - 0} = 66.73$ PF/m

EX: Coaxial conducting cylinders are located @ $\rho = 0.5$ & 1.2 cm.
The region between the cylinders is filled with perfect dielectric.
If the inner cylinder is at 100V, the outer at 0V.

a) The location of 20V equipotential sur.

b) $E_{\rho_{max}}$

c) ϵ_r if the charge per meter length on the inner cylinder 20 nC/m

$$V = f(r) \Longrightarrow \nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = 0$$

$$\Longrightarrow \frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = 0 \xrightarrow{\int} r\frac{\partial V}{\partial r} = A$$

$$\frac{\partial V}{\partial r} = \frac{A}{r} \xrightarrow{\int} V = A\ln r + B$$

$$V(0.005) = A\ln(0.005) + B = 100$$

$$V(0.012) = A\ln(0.012) + B = 0$$

$$\therefore A = -114.22$$

$$B = -505.2$$

$V = -114.22 \ln r - 505.2$

a) $20 = -114.22 \ln r - 505.2$

$\therefore r = 0.01 \text{ m} = 1 \text{ cm}$

b) $\bar{E} = -\nabla V = \frac{-dV}{dr} \hat{a}_r = \frac{114.22}{r} \hat{a}_r$

$\bar{E}_{max} (r = 0.005) = 22.84 \text{ KV/m}$

c) $\rho_{s_{+ve}} = |D|_{r=0.005} = \frac{114.22 * \epsilon_o * \epsilon_r}{0.005}$

$= 2.02 * 10^{-7} \epsilon_r \text{ C/m}^2$

$Q_{+ve} = \rho_{s_{+ve}} * 2\pi (0.005) L = 6.35 * 10^{-9} \epsilon_r$ C/m

$= 20 * 10^{-9}$

$\epsilon_r = 3.15$

* Sheet is very important

Question 2 : Final 2014-2015

a) $V = f(r)$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = 0 \Longrightarrow \frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = 0$$

$$\xrightarrow{\int} r\frac{\partial V}{\partial r} = A \Longrightarrow \frac{\partial V}{\partial r} = \frac{A}{r}$$

$$\xrightarrow{\int} V = A\ln r + B$$

$$\bar{E} = -\bar{\nabla} V = \frac{-\partial V}{\partial r}\hat{a}_r = \frac{-A}{r}\hat{a}_r$$

$$V(r=a) = A\ln a + B = 0 \longrightarrow 1$$

$$E(r=b) = \frac{-A}{b} = 1 \Longrightarrow A = -b \longrightarrow 2$$

From 2 in 1 $\Longrightarrow B = b\ln a$

$$V = -b\ln r + b\ln a$$

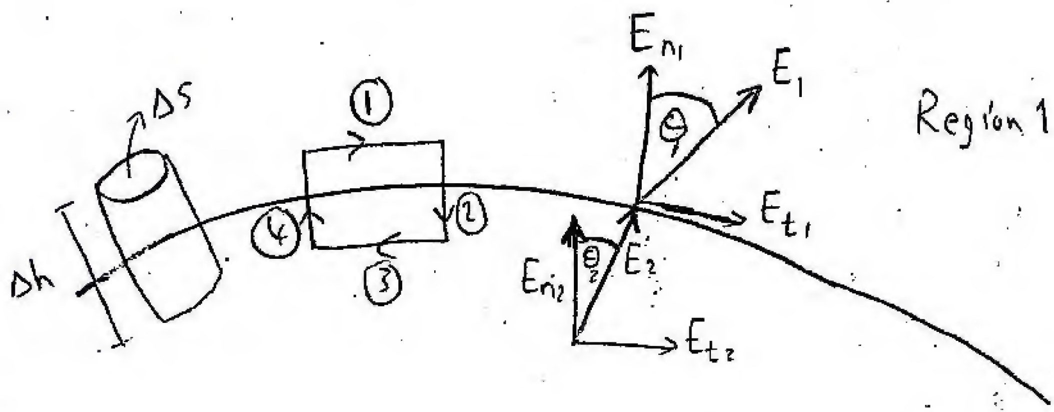
$$\bar{E} = \frac{b}{r}\hat{a}_r \Longrightarrow \bar{D} = \epsilon\bar{E} = \frac{\epsilon b}{r}\hat{a}_r$$

2) $\rho_s|_{in} = +|D|_{r=a} = \frac{\epsilon b}{a}$

$Q|_{in} = \rho_s|_{in} * 2\pi a L = 2\pi\epsilon b \quad C/m$

Dielectric- Dielectric Boundary Conditions

proof



Applying Gauss law

$$\oint \overline{D} \cdot \overline{ds} = Q_{en}$$

$$\int_{Top} \overline{D} \cdot \overline{ds} + \int_{bottom} \overline{D} \cdot \overline{ds} + \underbrace{\int_{side} \cancel{\overline{D}} \cdot \overline{ds}}_{\substack{0 \\ \overline{D} \perp \overline{ds}}} = \rho_s \Delta S$$

$$(D_{n_1} - D_{n_2}) \Delta S = \rho_s \Delta S$$

$$D_{n_1} - D_{n_2} = \rho_s$$

But the Interface is free of charges ($\rho_s = 0$)

$$\therefore D_{n_1} = D_{n_2}$$

Using $\oint \overline{E} \cdot \overline{dl} = 0$ and taking $\Delta h \simeq 0$

$$\therefore \int_1 \overline{E} \cdot \overline{dl} + \underbrace{\int_2 \overline{E} \cdot \overline{dl}}_{0 \ (\Delta h \simeq 0)} + \int_3 \overline{E} \cdot \overline{dl} + \underbrace{\int_4 \overline{E} \cdot \overline{dl}}_{0 \ (\Delta h \simeq 0)} = 0$$

$$E_{t_1} L + 0 + - E_{t_2} L + 0 = 0$$

$$\therefore E_{t_1} L - E_{t_2} L = 0$$

$$E_{t_1} = E_{t_2}$$

$\therefore$ Boundary conditions are

① $D_{n_1} = D_{n_2}$

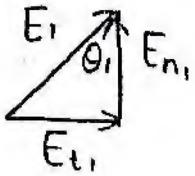
$\epsilon_1 E_{n_1} = \epsilon_2 E_{n_2}$

② $E_{t_1} = E_{t_2}$

$$\frac{D_{t_1}}{\epsilon_1} = \frac{D_{t_2}}{\epsilon_2}$$

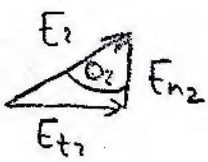
θ_1 = angle between (E_1 or D_1) with the direction normal to the interface

$$\theta_1 = \tan^{-1} \left| \frac{E_{t_1}}{E_{n_1}} \right| = \tan^{-1} \left| \frac{D_{t_1}}{D_{n_1}} \right|$$



θ_2 = angle between (E_2 or D_2) with the direction normal to the interface

$$\theta_2 = \tan^{-1} \left| \frac{E_{t_2}}{E_{n_2}} \right| = \tan^{-1} \left| \frac{D_{t_2}}{D_{n_2}} \right|$$



pb

Region ① $x > 0$, $\epsilon_{r_1} = 3$ $E_1 = 80\,\hat{a}_x - 60\,\hat{a}_y - 30\,\hat{a}_z$

Region ② $x < 0$, $\epsilon_{r_2} = 5$ Find E_2, D_1, D_2, θ_1, θ_2

Solution

$$E_{t_1} = -60\,\hat{a}_y - 30\,\hat{a}_z$$

$$D_{t_1} = \epsilon_1 E_{t_1}$$

$$D_{t_1} = -180\,\epsilon_0\,\hat{a}_y - 90\,\epsilon_0\,\hat{a}_z$$

$$E_{n_1} = 80\,\hat{a}_x$$

$$D_{n_1} = \epsilon_1 E_{n_1} = 240\,\epsilon_0\,\hat{a}_x$$

$$D_{n_1} = 240\,\epsilon_0\,\hat{a}_x$$

$\rightarrow E_{t_1} = E_{t_2}$

$$\therefore E_{t_2} = -60\,\hat{a}_y - 30\,\hat{a}_z$$

$$D_{t_2} = \epsilon_2 E_{t_2}$$

$$D_{t_2} = -300\,\epsilon_0\,\hat{a}_y - 150\,\epsilon_0\,\hat{a}_z$$

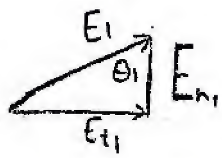
$\rightarrow D_{n_1} = D_{n_2}$

$$D_{n_2} = 240\,\epsilon_0\,\hat{a}_x$$

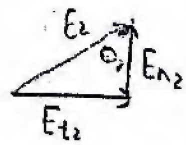
$$E_{n_2} = \frac{D_{n_2}}{\epsilon_2}$$

$$E_{n_2} = 48\,\hat{a}_x$$

$$* \ \theta_1 = \tan^{-1}\left|\frac{E_{t_1}}{E_{n_1}}\right| = \tan^{-1}\frac{\sqrt{60^2+30^2}}{80} = 39.98^\circ$$



$$* \ \theta_2 = \tan^{-1}\left|\frac{E_{t_2}}{E_{n_1}}\right| = \tan^{-1}\frac{\sqrt{60^2+30^2}}{48} = 54.4^\circ$$



pb. 2

Region 1 $y>0$, $\epsilon_{r_1}=2$ $\quad D_1 = 8\hat{a}_x + 10\hat{a}_y + 20\hat{a}_z$

Region 2 $y<0$, $\epsilon_{r_2}=5$, Find $E_1, D_2, E_2, \theta_1, \theta_2$

Solution

$$D_{n_1} = 10\hat{a}_y$$

$$E_{n_1} = \frac{D_{n_1}}{\epsilon_1} = \frac{D_{n_1}}{2\epsilon_0}$$

$$E_{n_1} = \frac{5}{\epsilon_0}\hat{a}_y$$

$$D_{t_1} = 8\hat{a}_x + 20\hat{a}_z$$

$$E_{t_1} = \frac{D_{t_1}}{\epsilon_{r_1}} = \frac{D_{t_1}}{2\epsilon_0}$$

$$E_{t_1} = \frac{4}{\epsilon_0}\hat{a}_x + \frac{10}{\epsilon_0}\hat{a}_z$$

$$D_{n_1} = D_{n_2}$$

$$D_{n_2} = 10\hat{a}_y$$

$$E_{n_2} = \frac{D_{n_2}}{\epsilon_2} = \frac{D_{n_2}}{5\epsilon_0}$$

$$E_{n_2} = \frac{2}{\epsilon_0}\hat{a}_y$$

$$E_{t_1} = E_{t_2}$$

$$E_{t_2} = \frac{4}{\epsilon_0}\hat{a}_x + \frac{10}{\epsilon_0}\hat{a}_z$$

$$D_{t_2} = \epsilon_2 E_{t_2} = 5\epsilon_0 E_{t_2}$$

$$D_{t_2} = 20\hat{a}_x + 50\hat{a}_z$$

$$* \ \theta_1 = \tan^{-1}\left|\frac{D_{t_1}}{D_{n_1}}\right| = \tan^{-1}\frac{\sqrt{8^2+20^2}}{10} = 65.09^\circ$$

$$* \ \theta_2 = \tan^{-1}\left|\frac{D_{t_2}}{D_{n_2}}\right| = \tan^{-1}\frac{\sqrt{20^2+50^2}}{10} = 79.48^\circ$$

Ex (Capacitance)

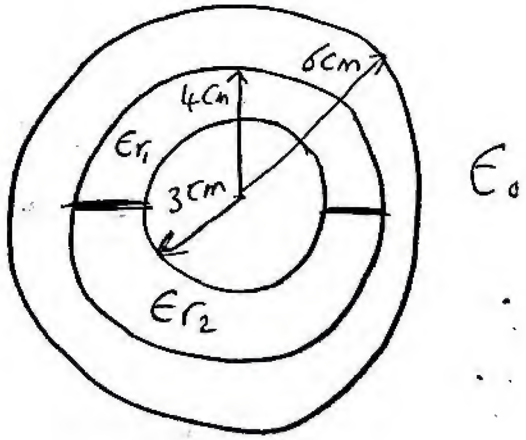
A small isolated conducting sphere of radius 3cm is charged with 0.3 μC. Surrounding this sphere and concentric with it a spherical shell which posses no net charge.

$\epsilon_{r_1} = 3$, $\epsilon_{r_2} = 5$, for $r > 6cm \Rightarrow$ Free space

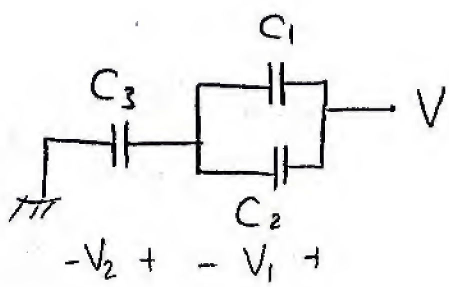
Find: (a) C_{eq}

(b) Absolute potential of each sphere

(c) Energy stored in the system



Solution



(a) $$C_1 = \frac{2\pi\epsilon_0\epsilon_{r1}}{\frac{1}{0.03} - \frac{1}{0.04}} = 2*10^{-11}\,F = 20\,pF$$

$$C_2 = \frac{2\pi\epsilon_0\epsilon_{r2}}{\frac{1}{0.03} - \frac{1}{0.04}} = 3.336*10^{-11} = 33.36\,pF$$

$$C_3 = \frac{4\pi\epsilon_0}{\frac{1}{0.06}} = 6.67*10^{-12}\,F = 6.67\,pF$$

$$C_{12} = C_1 + C_2 = 53.36\,pF$$

$$C_{eq} = \frac{C_3 C_{12}}{C_3 + C_{12}} = 5.93\,pF$$

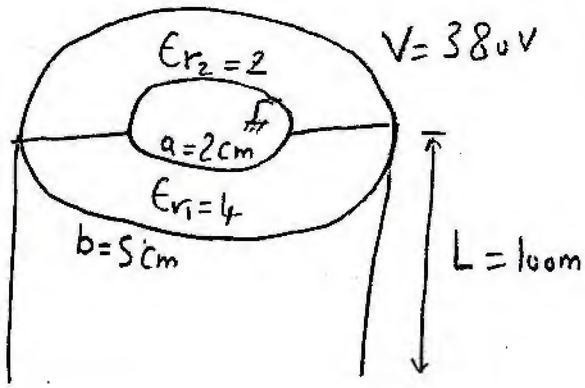
(b) $$C_{eq} = \frac{Q}{V} \Rightarrow V = 50.6\,KV \Rightarrow$$ Voltage of inner Conductor

$$C_3 = \frac{Q}{V_2} \Rightarrow V_2 = 44.98\,KV$$

(c) $$W_E = \frac{1}{2} C_{eq} V^2 = 7.6*10^{-3}\,J$$

EX

$V = 380$

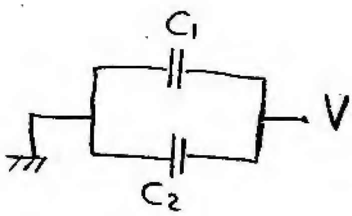


(a) $C_1 = \dfrac{\pi \epsilon_1 L}{\ln \frac{b}{a}} = \dfrac{\pi \epsilon_0 \epsilon_{r_1} L}{\ln \frac{b}{a}}$

$C_1 = \dfrac{\pi \epsilon_0 * 4 * 100}{\ln \frac{0.05}{0.02}} = 12.1 nF$

$C_2 = \dfrac{\pi \epsilon_2 L}{\ln \frac{b}{a}} = \dfrac{\pi \epsilon_0 \epsilon_{r_2} L}{\ln \frac{b}{a}} = \dfrac{\pi \epsilon_0 * 2 * 100}{\ln \frac{0.05}{0.02}} = 6.07 nF$

$C_{eq} = C_1 + C_2 = 18.2 nF$



(b) $W = \frac{1}{2} C_{eq} V^2$

$W = \frac{1}{2} * 18.2 * 10^{-9} * (380)^2$

$W = 1.31 * 10^{-3} J$

Magnetic part

Magnetic Revision

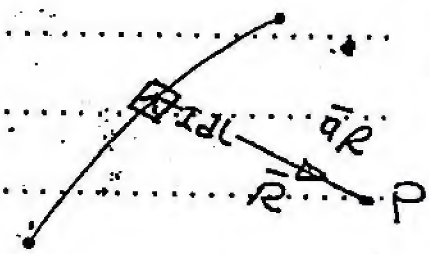
Part (B):

1 Bio_savart's law:

• Magnetic field intensity at any point (P):

(i) Due to filamentary Current:

$$\boxed{\bar{H} = \int \frac{\bar{I}\,dl}{4\pi R^2} \times \bar{a}_R}$$

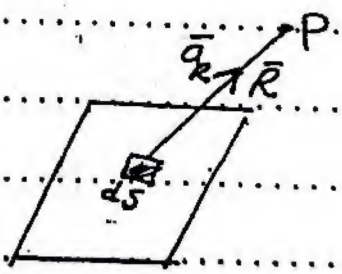


• R : Distance btn ($\bar{I}dl$) & (P)

• $\bar{a}_R$: unit vector of position vector ($\bar{R}$)

(ii) Surface Current

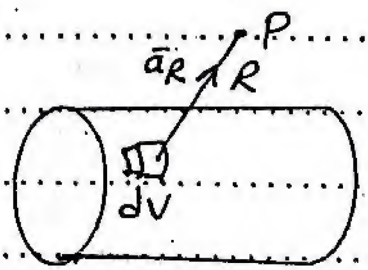
$$\bar{H} = \iint \frac{\bar{K}_s\,ds \times \bar{a}_R}{4\pi R^2}$$



• Ks : surface current density

(iii) Volumetric Current:

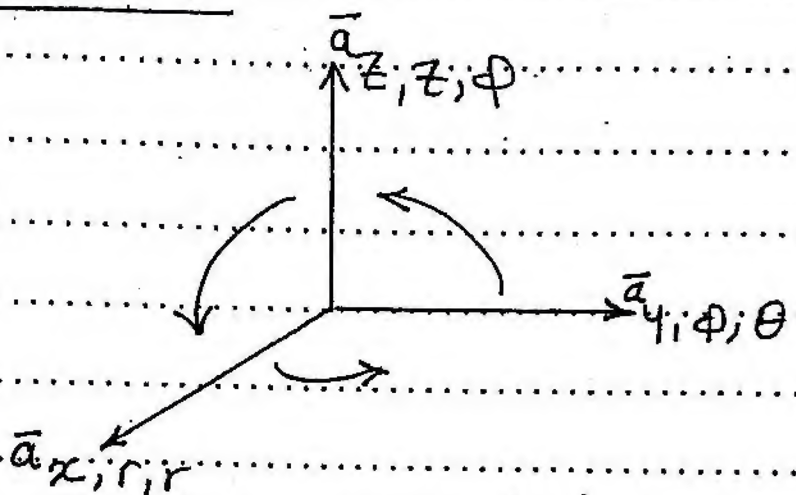
$$\boxed{\bar{H} = \iiint \frac{\bar{J}_v\,dV \times \bar{a}_R}{4\pi R^2}}$$



• Jv : volume current density

- All Current elements ($\bar{I}\,dl$, $\bar{K}_s\,ds$ & $\bar{J}_v\,dv$) have the direction of the current.

- **Cross product:**



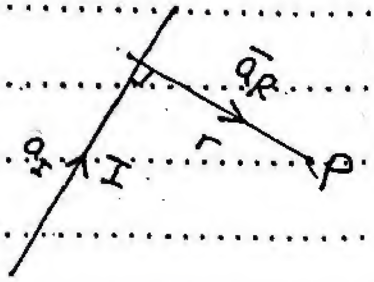
- **For any problem:** (except lines)

1. Find $\bar{I}\,dl$ or $\bar{K}_s\,ds$ or $J_v\,dv$
2. Find $\bar{R} \Rightarrow |\bar{R}| \Rightarrow \bar{a}_R$
3. check for symmetry via "R.H.R"
4. Integrate

- **For Filamentary wires:** فيديو

(i) **Infinite wire:**

$$\boxed{\bar{H} = \frac{I}{2\pi r}\,(\bar{a}_I \times \bar{a}_R)}$$



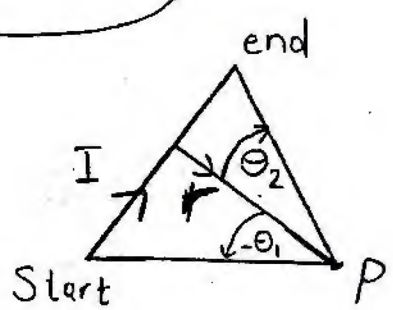
- $\bar{a}_I$: Current's unit vector

$\bar{a}_R$: Unit vector of the normal from p to the wire and directed towards p

(ii) Finite wire

$$\boxed{\overline{H} = \frac{I}{4\pi r}\left(\sin\theta_2 - \sin\theta_1\right)\left[\hat{a}_I \times \hat{a}_R\right]}$$

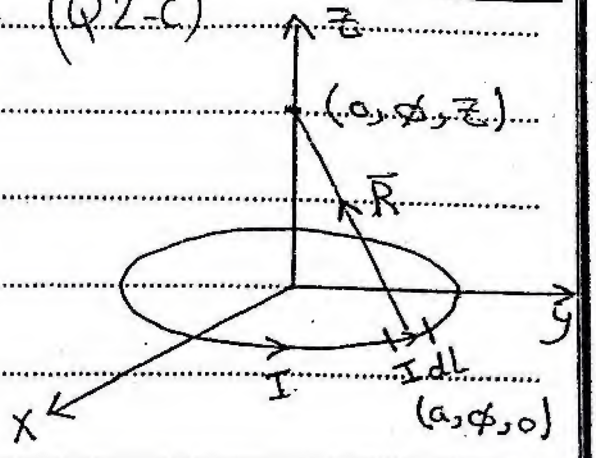
- $\hat{a}_I, \hat{a}_R, r$: Same as infinite wire



- θ_2 : Angle between r and the end of the line in clock-wise direction
- θ_1 : Angle between r and the start of the line in clock-wise direction
- $\frac{I}{4\pi r}\left(\sin\theta_2 - \sin\theta_1\right)$ is magnitude of H

So, If it's negative take the modulus

pb ① : Midterm 2014-2015 (Q2-c)



$dL = a\, d\phi$

$\bar{I}\, dL = I a\, d\phi\, \hat{a}_\phi$

$\bar{R} = -a\, \hat{a}_r + z\, \hat{a}_z$

$|R| = \sqrt{a^2 + z^2}$

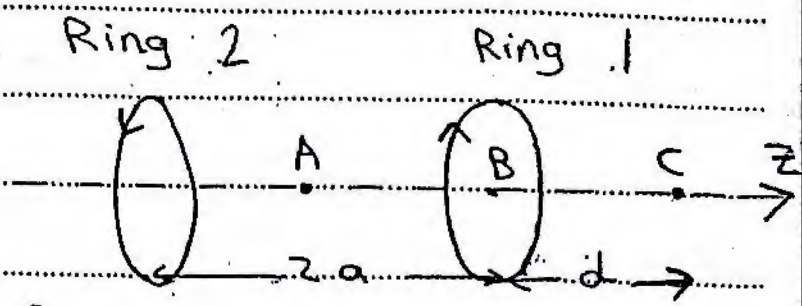
$$\hat{a}_R = \frac{-a\,\hat{a}_r + z\,\hat{a}_z}{\sqrt{a^2+z^2}}$$

$$d\bar{H} = \frac{\bar{I}dL \times \hat{a}_r}{4\pi R^2} = \frac{I a\, d\phi}{4\pi (a^2+z^2)^{3/2}} \left\{ \hat{a}_\phi \times (-a\,\hat{a}_r + z\,\hat{a}_z) \right\}$$

$$d\bar{H} = \frac{I a\, d\phi}{4\pi (a^2+z^2)^{3/2}} \left[a\,\hat{a}_z + z\,\hat{a}_r \right].$$

From R.H.R $\quad H_r = 0$

$$H = \int_0^{2\pi} \frac{I a^2\, d\phi}{4\pi (a^2+z^2)^{3/2}}\, \hat{a}_z = \frac{I a^2}{2 (a^2+z^2)^{3/2}}\, \hat{a}_z$$



Ring 1 $\quad \bar{H} = \dfrac{I a^2}{2(a^2+z_1^2)^{3/2}} \hat{a}_z$

Ring 2 $\quad \bar{H} = \dfrac{I a^2}{2(a^2+z_2^2)^{3/2}} (-\hat{a}_z)$

1) $z_1 = a \quad , \quad z_2 = a$

$$H_A = \frac{I a^2}{2(a^2+a^2)^{3/2}} \hat{a}_z + \frac{I a^2}{2(a^2+a^2)^{3/2}} -\hat{a}_z = 0 \text{ AT/m}$$

2) $z_1 = 0 \qquad z_2 = 2a$

$$H_B = \frac{I a^2}{2(a^2+0)^{3/2}} \hat{a}_z + \frac{I a^2}{2(a^2+4a^2)^{3/2}} -\hat{a}_z = \frac{0.455 I}{a} \hat{a}_z$$

3) $z_1 = d \qquad z_2 = d + 2a \simeq d \qquad (d >> a)$

$\therefore H_c = 0$ AT/m

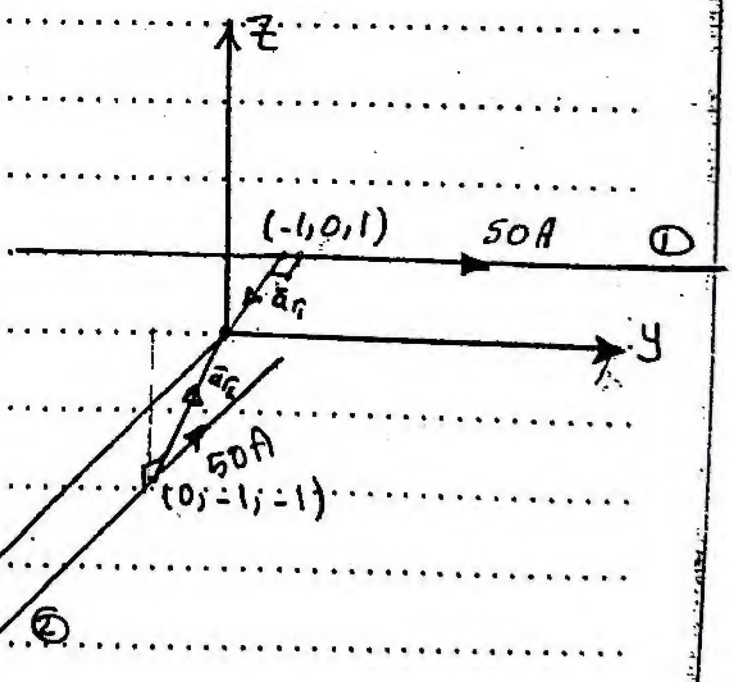
Pb.(2): final (2013 – 2014)

Given: Infinite wire (1): 50 A in (+ve) y-direction & passes through (−1, 0, 1)

Infinite wire (2): 50 A in (−ve) x-direction & passes through (0, −1, −1)

Find: H at the origin

Solution:



$\bar{H}_o = \bar{H}_1 + \bar{H}_2$

$\therefore \bar{H}_1 = \frac{I}{2\pi r_1} (\bar{a}_I \times \bar{a}_{r_1})$

$\bar{r}_1 = (0,0,0) - (-1,0,1)$

$\therefore \bar{r}_1 = 1\bar{a}_x - 1\bar{a}_z$

$\therefore r_1 = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$\therefore \bar{a}_{r_1} = \frac{1\bar{a}_x - 1\bar{a}_z}{\sqrt{2}}$

$\therefore \bar{H}_1 = \frac{50}{2\pi\sqrt{2}} (\bar{a}_y) \times \left(\frac{\bar{a}_x - \bar{a}_z}{\sqrt{2}}\right)$

$\therefore \boxed{\bar{H}_1 = 3.98(-\bar{a}_z - \bar{a}_x)}$

$\therefore \bar{H}_2 = \frac{I}{2\pi r_2} (\bar{a}_I \times \bar{a}_{r_2})$

$\bar{r}_2 = (0, 0, 0) - (0, -1, -1) = \bar{a}_y + \bar{a}_z$

$\therefore r_2 = \sqrt{1^2 + 1^2} = \sqrt{2}$

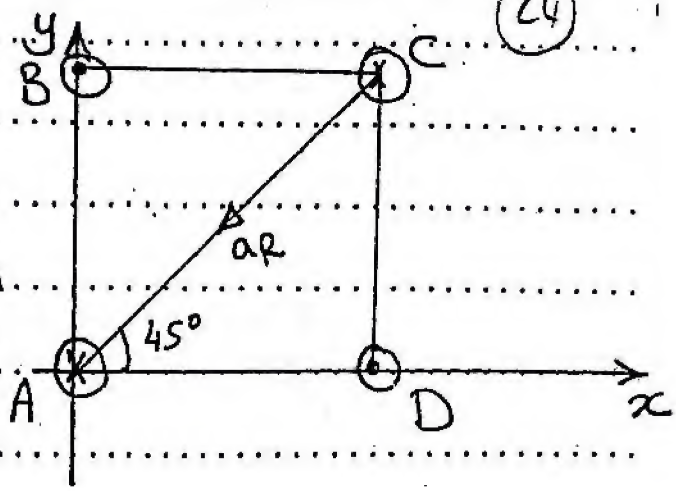
$\therefore \bar{a}_{r_2} = \frac{\bar{a}_y + \bar{a}_z}{\sqrt{2}}$

$\therefore \bar{H}_2 = \frac{50}{2\pi\sqrt{2}} (-\bar{a}_x) \times \left(\frac{\bar{a}_y + \bar{a}_z}{r_2}\right)$

$\therefore \boxed{\bar{H}_2 = 3.98 (-\bar{a}_z + \bar{a}_y)}$

$\therefore \boxed{\bar{H}_o = -3.98\,\bar{a}_x + 3.98\,\bar{a}_y - 7.96\,\bar{a}_z}$

Pb (3)



Given:

$AB = BC = CD = AD = 0.1\,m$

$I_A = I_B = I_C = I_D = 200\,A$

Find: $\bar{H}$ @ A

Soln:

$$\bar{H}_A = \bar{H}_B + \bar{H}_C + \bar{H}_D$$

$$\therefore \bar{H}_B = \frac{I}{2\pi r}(\bar{a}_I \times \bar{a}_R) = \frac{200}{2\pi(0.1)}((+\bar{a}_z)\times(-\bar{a}_y))$$

$$\boxed{\therefore \bar{H}_B = 318.3\,\bar{a}_x}$$

$$\therefore \bar{H}_C = \frac{I}{2\pi r}(\bar{a}_I \times \bar{a}_R) = \frac{200}{2\pi(0.1\sqrt{2})}(-\bar{a}_z \times \{-\cos 45\,\bar{a}_x - \sin 45\,\bar{a}_y$$

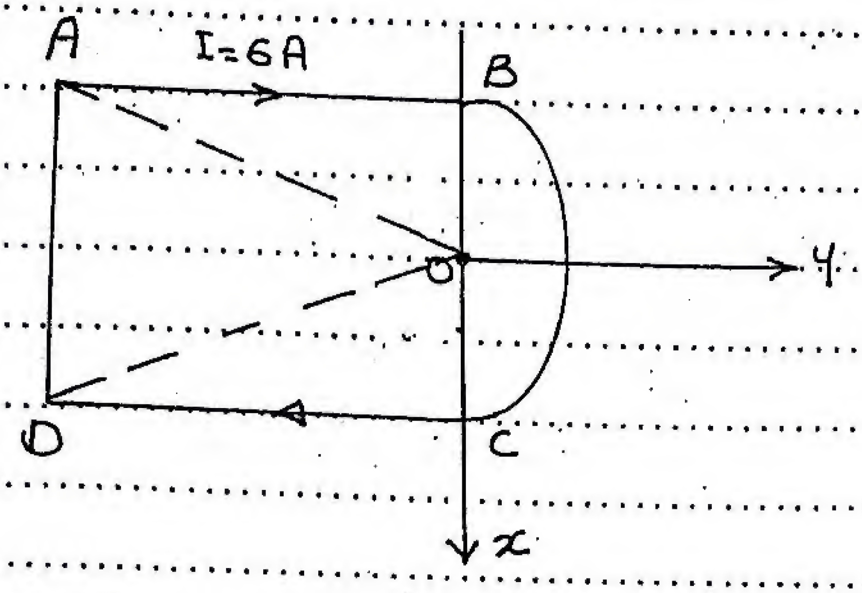
$$\boxed{\therefore \bar{H}_C = 159.1(\bar{a}_y - \bar{a}_x)}$$

$$\therefore \bar{H}_D = \frac{I}{2\pi r}(\bar{a}_I \times \bar{a}_R) = \frac{200}{2\pi(0.1)}(\bar{a}_z \times -\bar{a}_x)$$

$$\boxed{\therefore \bar{H}_D = -318.3\,\bar{a}_y}$$

$$\therefore \boxed{\bar{H}_A = 159.2\,\bar{a}_x - 159.2\,\bar{a}_y}$$

Pb(4)



$AB = CD = 2\,m$, $AD = 0.1\,m$, $BC = 0.1\,m$

$I = 6A$

Find : $\bar{H}$ at "O"

Solution:

$$\therefore \bar{H}_O = \bar{H}_{AB} + \bar{H}_{BC} + \bar{H}_{CD} + \bar{H}_{AD}$$

$$\therefore \bar{H}_{AB} = \frac{I}{4\pi r}(\sin\theta_2 - \sin\theta_1)(\bar{a}_I \times \bar{a}_R)$$

$\therefore r = \frac{BC}{2} = \frac{0.1}{2}\,m$, $\theta_2 = 0$, $\theta_1 = \tan^{-1}\frac{2}{\frac{0.1}{2}} = -88.5°$

$\bar{a}_I = \bar{a}_y$, $\bar{a}_R = \bar{a}_x$

anti-clock wise

$$\boxed{\therefore \bar{H}_{AB} = -9.55\,\bar{a}_z}$$

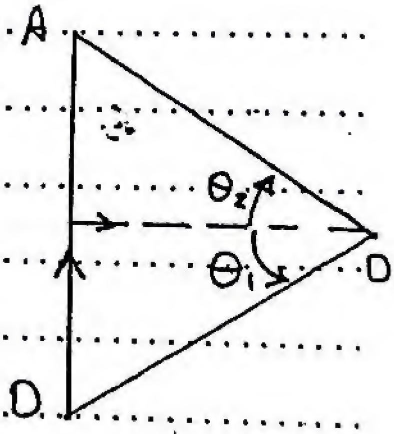
$$\bar{H}_{CD} = \frac{I}{4\pi r}(\sin\theta_2 - \sin\theta_1)(\bar{a}_I \times \bar{a}_R)$$ (26)

$\therefore r = \frac{0.1}{2}$ m , $\theta_1 = 0$, $\theta_2 = \tan^{-1}\left(\frac{2}{\frac{0.1}{2}}\right) = 88.5$

$a_I = -\bar{a}_y$, $\bar{a}_R = -\bar{a}_x$

$$\therefore \boxed{\bar{H}_{CD} = -9.55\,\bar{a}_z}$$

$$\therefore \bar{H}_{AD} = \frac{I}{4\pi r}(\sin\theta_2 - \sin\theta_1)(\bar{a}_I \times \bar{a}_R)$$

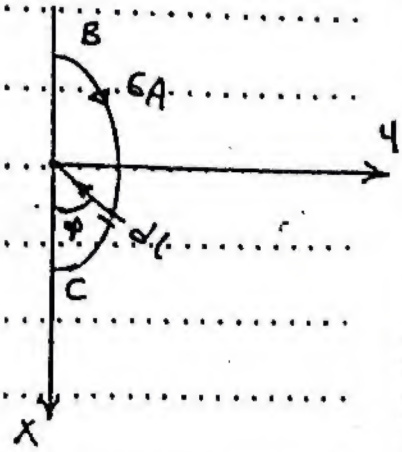


$\theta_2 = \tan^{-1}\frac{0.1/2}{2} = 1.43°$

$\theta_1 = -\tan^{-1}\frac{0.1/2}{2} = -1.43$

$\bar{a}_I = -\bar{a}_x$, $\bar{a}_R = \bar{a}_y$, $r = 2$m

$$\therefore \boxed{\bar{H}_{AD} = 0.012\,(-\bar{a}_z)}$$



$$\bar{H}_{BC} = \int \frac{\bar{I}\,dl \times \bar{a}_R}{4\pi R^2}$$

$\bar{I}\,dl = 6 * \frac{0.1}{2}\, d\phi\,(-\bar{a}_\phi)$

$\bar{a}_R = -\bar{a}_r$

$R = \frac{0.1}{2} = 0.05$

$$\therefore H_{BC} = \int_0^{\pi} \frac{6 \times \frac{0.1}{2} \, d\phi \, (-\bar{a}_\phi) \times (-\bar{a}_R)}{4\pi (0.05)^2}$$

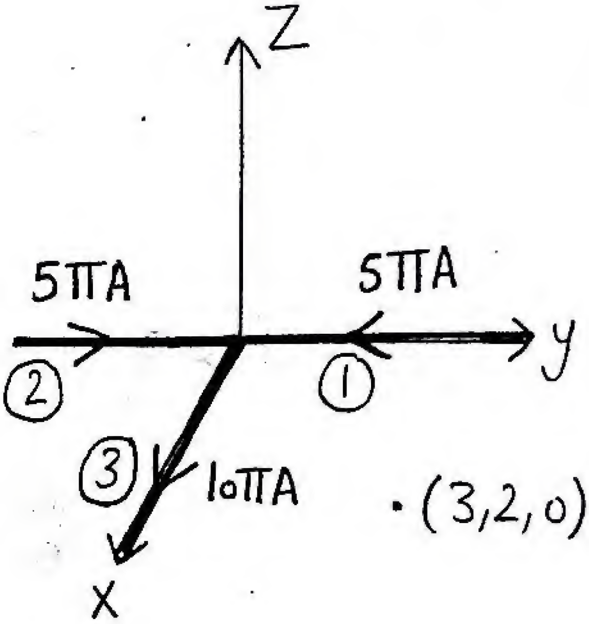
$$\therefore \boxed{\bar{H}_{BC} = 30 \, (-\bar{a}_z)}$$

$$\therefore \boxed{\bar{H}_o = 49.08 \, (-\bar{a}_z)}$$

pb. 5

Find $\overline{H}$ at $(3,2,0)$

$$\overline{H} = \overline{H_1} + \overline{H_2} + \overline{H_3}$$

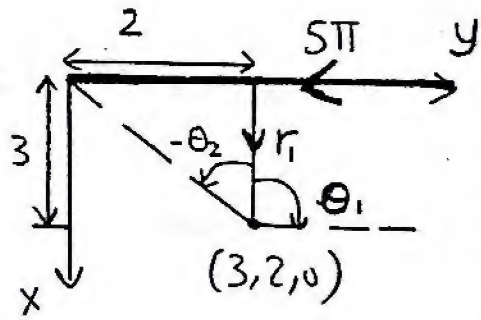


$\Rightarrow$ H_1

$\hat{a}_I = -\hat{a}_y$ $\hat{a}_{r_1} = \hat{a}_x$

- $r_1 = 3$
- $\theta_1 = 90$
- $\theta_2 = -\tan^{-1}\frac{2}{3} = -33.69$

$$\therefore H_1 = \frac{I}{4\pi r_1}\left(\sin\theta_2 - \sin\theta_1\right)\left(a_I \times a_{r_1}\right)$$



$$H_1 = \underbrace{\frac{5\pi}{4\pi(3)}\left(-0.55-1\right)}_{\text{mag. of } H \text{ (+ve)}}\left(-a_y \times a_x\right)$$

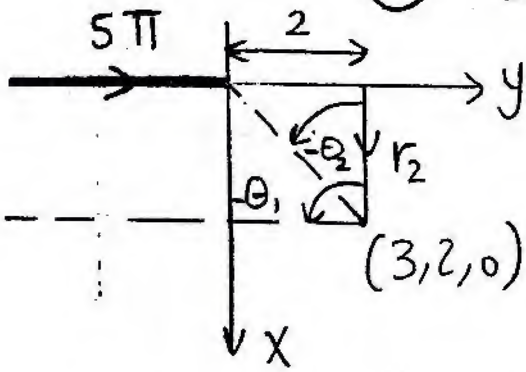
$(+a_z)$

$$\boxed{\therefore H_1 = 0.646(a_z)}$$ $\rightarrow$ We take the modulus of the magnitude

(29)

⟹ H_2

$\hat{a}_I = \hat{a}_y$ $\hat{a}_{r_2} = \hat{a}_x$



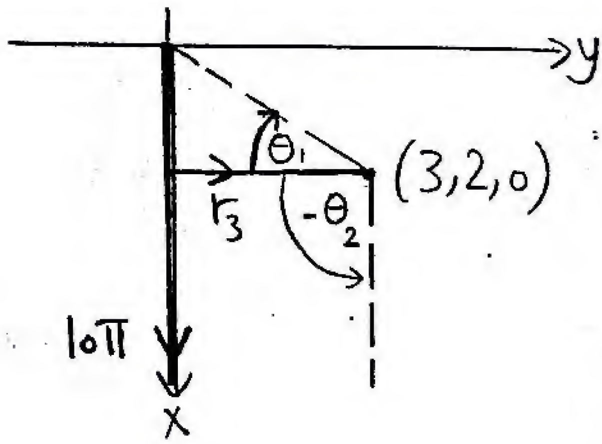
- $r_2 = 3$
- $\theta_1 = -90$
- $\theta_2 = -33.69$

$$\overline{H}_2 = \frac{5\pi}{4\pi(3)} (-0.55+1)(\hat{a}_y \times \hat{a}_x)$$

$$\overline{H}_2 = 0.1875(-\hat{a}_z)$$

⟹ $\overline{H}_3$

$\hat{a}_I = \hat{a}_x$ $\hat{a}_{r_3} = \hat{a}_y$



- $r_3 = 2$
- $\theta_1 = \tan^{-1}\frac{3}{2} = 56.31$
- $\theta_2 = -90°$

$$\overline{H}_3 = \frac{10\pi}{4\pi(2)} (\sin\theta_2 - \sin\theta_1)(\hat{a}_x \times \hat{a}_y)$$

$$H_3 = 2.29 \quad (a_z)$$

→ we take the modulus of the magnitude

$$\overline{H} = \overline{H}_1 + \overline{H}_2 + \overline{H}_3$$

$$\therefore H = 2.75 \ a_z \ A/m$$

Final 2016 : Q2

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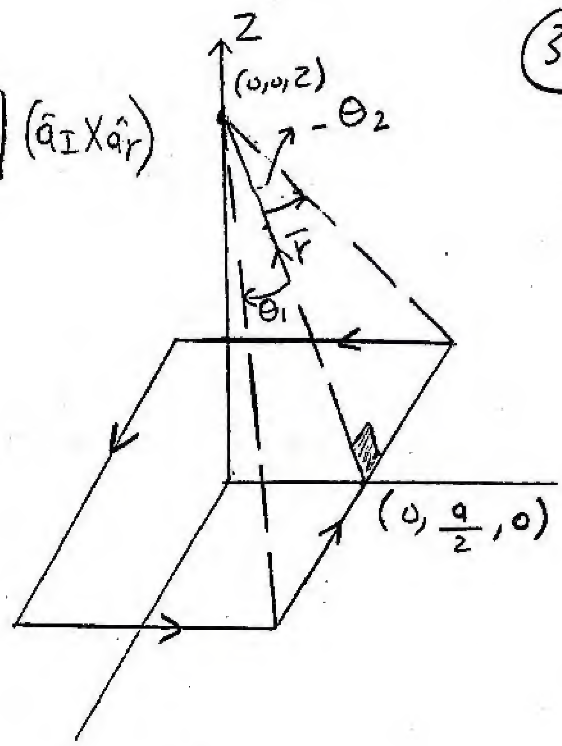


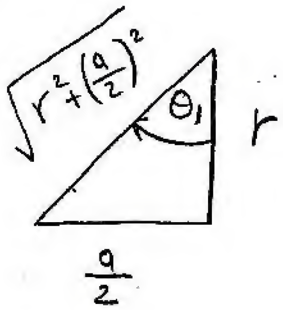
$$H = \frac{I}{4\pi r}\left[\sin\theta_2 - \sin\theta_1\right](\hat{a}_I \times \hat{a}_r)$$

$$* \ \bar{r} = \frac{-a}{2}\hat{a}_y + Z\hat{a}_z$$

$$|\bar{r}| = \sqrt{\left(\frac{a}{2}\right)^2 + Z^2}$$

$$\hat{a}_r = \frac{\frac{-a}{2}\hat{a}_y + Z\hat{a}_z}{\sqrt{\left(\frac{a}{2}\right)^2 + Z^2}}$$

$$* \ \sin\theta_1 = \frac{\frac{a}{2}}{\sqrt{r^2 + \left(\frac{a}{2}\right)^2}}$$



$$\sin\theta_1 = \frac{\frac{a}{2}}{\sqrt{\left(\frac{a}{2}\right)^2 + Z^2 + \left(\frac{a}{2}\right)^2}}$$

$$\sin\theta_1 = \frac{\frac{a}{2}}{\sqrt{\frac{a^2}{2} + Z^2}} = \frac{a}{\sqrt{4Z^2 + 2a^2}}$$

$$\therefore \sin\theta_1 = \frac{a}{\sqrt{4Z^2 + 2a^2}}$$

$$* \ \sin\theta_2 = \frac{-a}{\sqrt{4Z^2 + 2a^2}}$$

Magnitude of H

$$|H| = \left| \frac{I}{4\pi r} \left[\sin\theta_2 - \sin\theta_1 \right] \right|$$

$$= \left| \frac{I}{4\pi \sqrt{\left(\frac{a}{2}\right)^2 + Z^2}} \left[\frac{-a}{\sqrt{4Z^2+2a^2}} - \frac{a}{\sqrt{4Z^2+2a^2}} \right] \right|$$

$$|H| = \frac{I}{4\pi \sqrt{\left(\frac{a}{2}\right)^2 + Z^2}} * \frac{2a}{\sqrt{4Z^2+2a^2}}$$ → for 1 side

Direction of $\overline{H}$

$$a_I \times \hat{a}_r = -\hat{a}_x \times \frac{\frac{-a}{2}\hat{a}_y + Z\hat{a}_z}{\sqrt{\left(\frac{a}{2}\right)^2 + Z^2}}$$

$$= \frac{\frac{a}{2}\hat{a}_z + Z\hat{a}_y}{\sqrt{\left(\frac{a}{2}\right)^2 + Z^2}}$$

According to R.H.R $\Rightarrow \hat{a}_I \times \hat{a}_r = \frac{\frac{a}{2}\hat{a}_z}{\sqrt{\left(\frac{a}{2}\right)^2 + Z^2}}$

$$\therefore H_{1\,side} = \left(\frac{I}{4\pi \sqrt{\left(\frac{a}{2}\right)^2 + Z^2}} * \frac{2a}{\sqrt{4Z^2+2a^2}} \right) \left(\frac{\frac{a}{2}\hat{a}_z}{\sqrt{\left(\frac{a}{2}\right)^2 + Z^2}} \right)$$

$$H_{1\,side} = \frac{I a^2}{4\pi \sqrt{4Z^2 + 2a^2}} \cdot \frac{1}{\left(Z^2 + \left(\frac{a}{2}\right)^2\right)} (\hat{a}_z)$$

32

$$\overline{H}_{total} = 4\,\overline{H}_{1 side} = \frac{4 I a^2}{4\pi\sqrt{4Z^2+2a^2}} \, \frac{1}{\left(Z^2+\left(\frac{a}{2}\right)^2\right)} \, \hat{a}_z$$

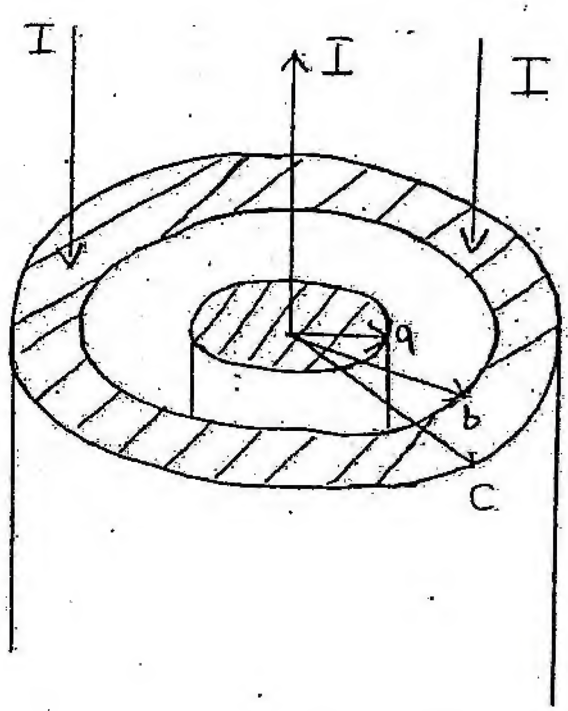
$$H_{total} = \frac{4 I a^2}{\pi\sqrt{4Z^2+2a^2}\,\left(4Z^2+a^2\right)} \, \hat{a}_z$$

$$B_{total} = \frac{2\sqrt{2}\, I a^2 \mu}{\pi\left(4Z^2+a^2\right)\sqrt{2Z^2+a^2}} \left(\hat{a}_z\right)$$

Ex: Use Ampere's law to find $\overline{H}$ in the range of $0 < \rho < \infty$ in the shown Coaxial Cable arrangement.

Also sketch $|\overline{H}|$ Versus ρ

Solution



$\Rightarrow$ for $\rho < a$ (region I)

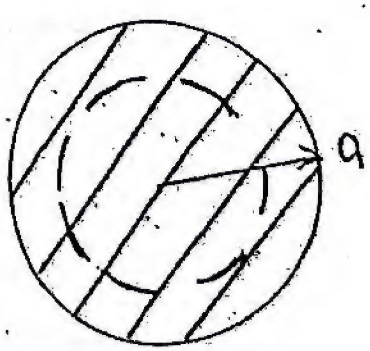
$$\oint \overline{H} \cdot \overline{dl} = I_{en}$$

$$I_{en} = J_1 * \pi \rho^2$$

Where $J_1 = \frac{I}{\pi a^2}$

$$\therefore H * 2\pi\rho = \frac{I}{\pi a^2} * \pi \rho^2$$

$$\therefore \overline{H} = \frac{I\rho}{2\pi a^2} a_\phi$$



$\rightarrow$ at $\rho = 0 \Rightarrow H = 0$

$\rightarrow$ at $\rho = a \Rightarrow H = \frac{I}{2\pi a}$

⇒ for $a < \rho < b$ (region II)

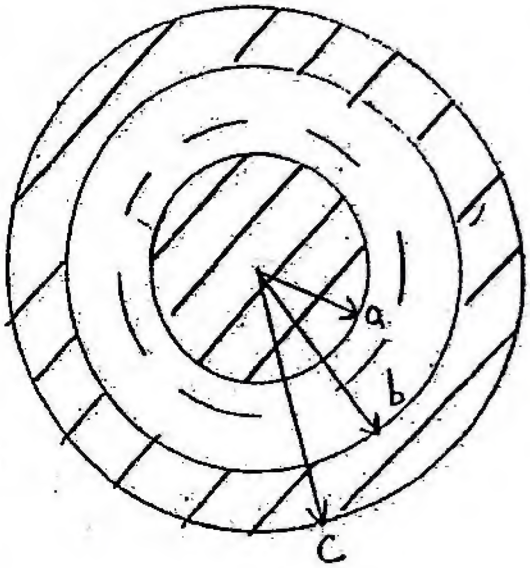
$$\oint \overline{H} \cdot \overline{dl} = I_{en}$$

$$H * 2\pi\rho = I$$

$$\overline{H} = \frac{I}{2\pi\rho} \, a_\phi$$

→ at $\rho = a$ ⇒ $H = \frac{I}{2\pi a}$

→ at $\rho = b$ ⇒ $H = \frac{I}{2\pi b}$



⇒ for $b < \rho < c$ (region III)

$$\oint \overline{H} \cdot \overline{dl} = I_{en}$$

$$H * 2\pi\rho = I - J_2 * \pi(\rho^2 - b^2)$$, where $J_2 = \frac{I}{\pi(c^2 - b^2)}$

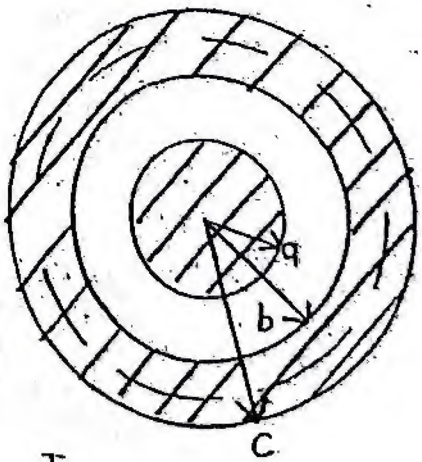
$$\therefore H * 2\pi r = I - \frac{I * \pi(\rho^2 - b^2)}{\pi(C^2 - b^2)}$$

$$H * 2\pi r = I \left[\frac{c^2 - \rho^2}{c^2 - b^2} \right]$$

$$\therefore \overline{H} = \frac{I}{2\pi r} \left[\frac{C^2 - \rho^2}{c^2 - b^2} \right] a_\phi$$

→ at $\rho = b$ ⇒ $H = \frac{I}{2\pi b}$

→ at $\rho = c$ ⇒ $H = 0$

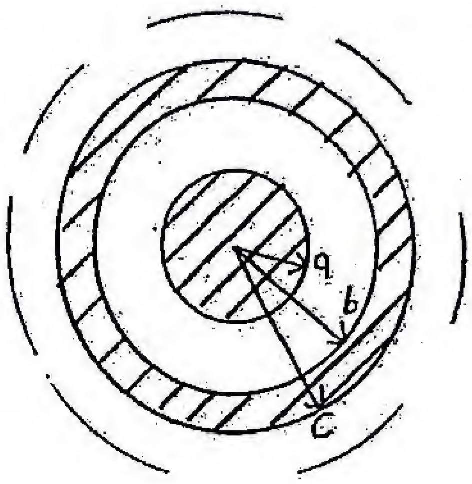


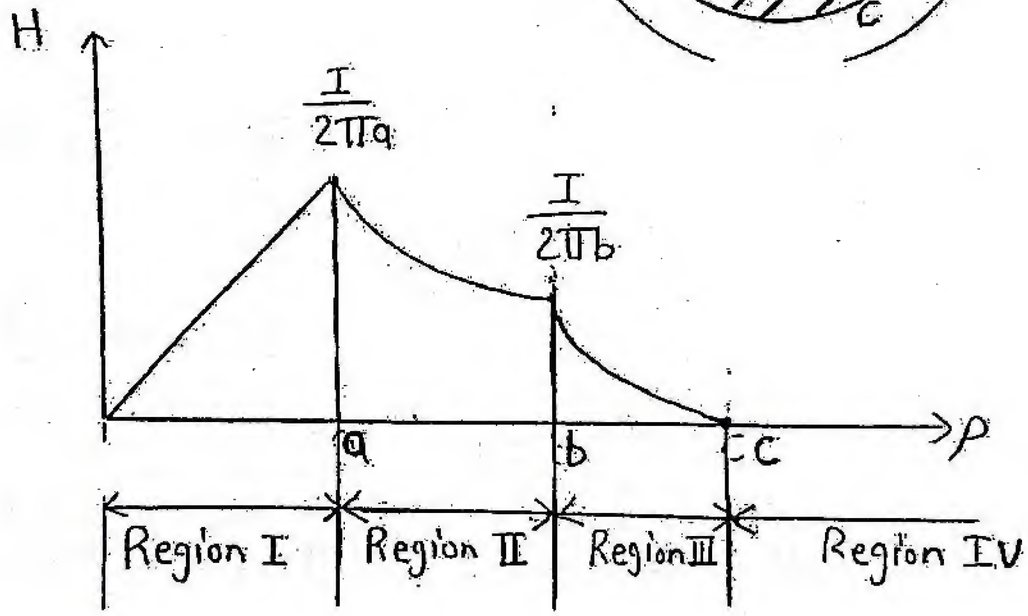
⇒ For $\rho > c$ (region IV)

$$\oint \overline{H} \cdot \overline{dL} = I_{en}$$

$$H * 2\pi\rho = I - I = 0$$

$$\therefore H = 0$$





$\therefore$ For Coaxial Cables

- $\overline{H} = \dfrac{I\rho}{2\pi a^2}\,\hat{a}_\phi$ For $(a<\rho<a)$
- $\overline{H} = \dfrac{I}{2\pi\rho}\,\hat{a}_\phi$ For $(a<\rho<b)$
- $\overline{H} = \dfrac{I}{2\pi\rho}\left[\dfrac{c^2-\rho^2}{c^2-b^2}\right]\hat{a}_\phi$ For $(b<\rho<c)$
- $\overline{H} = 0$ For $\rho > c$

Magnetic flux density and Magnetic flux

- $B \rightarrow$ Magnetic flux density (Wb/m^2 or Tesla)
- $\Phi \rightarrow$ Magnetic flux (Wb)
- $H \rightarrow$ Magnetic flux intensity (A/m)

Relations

① $\overline{B} = \mu \overline{H}$

Where $\mu = \mu_o \mu_r$ (μ_r : relative permittivity)

② $\int \overline{B} \cdot \overline{ds} = \Phi$

EX ② For the Coaxial cable in EX ①, Find the magnetic Flux per unit length.

Solution

$$\Phi = \int \overline{B} \cdot \overline{ds}$$

For Coaxial cable ($B = \mu_o H$):

$$\overline{B} = \frac{\mu_o I \rho}{2\pi a^2} \hat{a}_\phi \quad (0 < \rho < a)$$

$$\overline{B} = \frac{\mu_o I}{2\pi \rho} \hat{a}_\phi \quad (a < \rho < b)$$

$$\overline{B} = \frac{\mu_o I}{2\pi \rho} \left[\frac{c^2 - \rho^2}{c^2 - b^2}\right] \quad (b < \rho < c)$$

$$B = 0 \quad (\rho > c)$$

$\Longrightarrow$ For $0 < \rho < a$

$$\Phi = \int \overline{B} \cdot \overline{ds}$$

$$= \int_0^L \int_0^a \frac{\mu_0 I \rho}{2\pi a^2} * d\rho \, dZ$$

$$= \frac{\mu_0 I}{2\pi a^2} \left[\frac{\rho^2}{2} \Big|_0^a \right] \left[Z \Big|_0^L \right]$$

$$\therefore \Phi = \frac{\mu_0 I}{4\pi} L$$

$$\therefore \frac{\Phi}{L} = \frac{\mu_0 I}{4\pi} \quad Wb/m$$

$\Longrightarrow$ For $a < \rho < b$

$$\Phi = \int \overline{B} \cdot \overline{ds}$$

$$= \int_0^L \int_a^b \frac{\mu_0 I}{2\pi \rho} * d\rho \, dZ$$

$$= \frac{\mu_0 I}{2\pi} \left[\ln\rho \Big|_a^b \right] \left[Z \Big|_0^L \right]$$

$$\therefore \Phi = \frac{\mu_0 I}{2\pi} \ln\left(\frac{b}{a}\right) * L$$

$$\frac{\Phi}{L} = \frac{\mu_0 I}{2\pi} \ln\left(\frac{b}{a}\right) \quad (Wb/m)$$

$\Longrightarrow$ For $(b<\rho<c)$

$$\Phi = \int \overline{B} \cdot \overline{ds}$$

$$= \int_0^L \int_b^c \frac{\mu_0 I}{2\pi\rho} \left[\frac{c^2 - \rho^2}{c^2 - b^2} \right] * d\rho \, dZ$$

$$= \int_0^L \int_b^c \frac{\mu_0 I}{2\pi(c^2-b^2)} \left[\frac{c^2}{\rho} - \rho \right] d\rho \, dZ$$

$$= \frac{\mu_0 I}{2\pi(c^2-b^2)} \left[c^2 \ln\rho \Big|_b^c - \frac{\rho^2}{2} \Big|_b^c \right] \left[Z \Big|_0^L \right]$$

$$= \frac{\mu_0 I}{2\pi(c^2-b^2)} \left[c^2 \ln\frac{c}{b} - \frac{(c^2-b^2)}{2} \right] * L$$

$$\therefore \frac{\Phi}{L} = \frac{\mu_0 I}{2\pi(c^2-b^2)} \left[c^2 \ln\left(\frac{c}{b}\right) - \left(\frac{c^2-b^2}{2}\right) \right]$$

$\Longrightarrow$ For $(\rho>c)$

$$\Phi = \int \overline{B} \cdot \overline{ds}$$

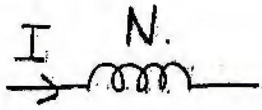
$$= \int 0 \cdot \overline{ds}$$

$$= 0$$

$$\frac{\Phi}{L} = 0$$

Inductance

Consider a coil with N-turns carrying a current $I \Rightarrow I$ produces a flux ϕ

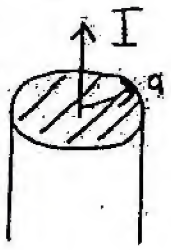


→ λ = flux linkage = $N\phi$ (weber.turn)

→ L = Inductance = $\frac{\lambda}{I} = \frac{N\phi}{I}$ (Henry)

Note

If we have a conductor carrying a current I.



∴ For $\rho > a$ (outside the conductor)

$N = 1$

For $\rho < a$ (Inside the conductor)

$N = \left(\frac{\rho}{a}\right)^2$

Steps to calculate the inductance of any Configuration

1. Find $\overline{H}$ using Ampere's law

2. Find $B \rightarrow \overline{B} = \mu \overline{H}$

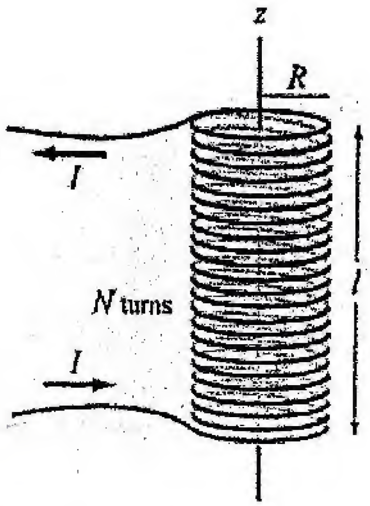
3. Find $\Phi \rightarrow \Phi = \int \overline{B} \cdot \overline{ds}$

4. $L = \dfrac{N\Phi}{I}$

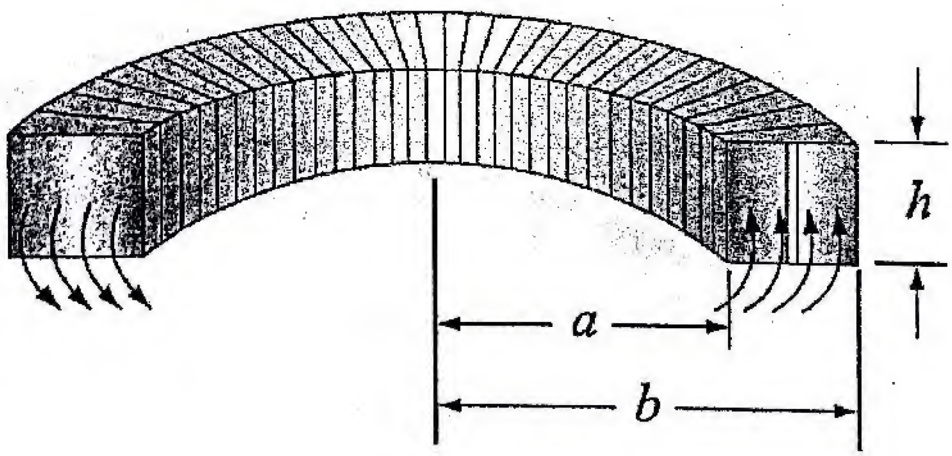
<u>Note:</u> The inductance depends only on dimensions of the Configuration

Question ٢

١) Compute the self-inductance of a solenoid with N turns, length L, and radius R with a current I flowing through each turn, as shown in the Figure.



٢) Calculate the self-inductance of a toroid which consists of N turns and has a rectangular cross section, with inner radius a, outer radius b and height h, as shown in the Figure.

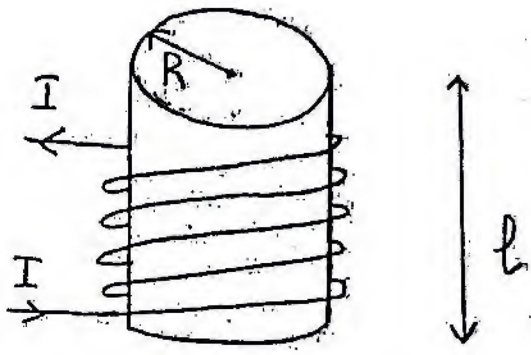


Sheet 3b

Question (2)

(1)

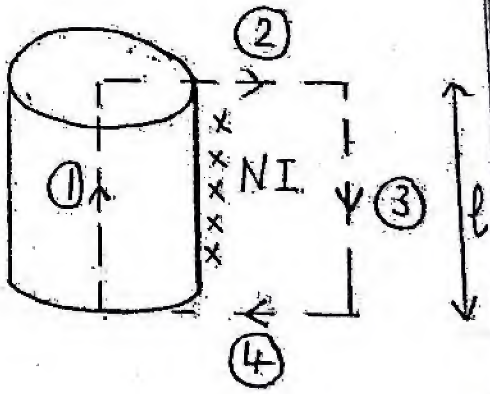
Step (1) Find H



using Ampere's law

$$\oint \overline{H} \cdot \overline{dl} = I_{en}$$

$$\int_{1} \overline{H} \cdot \overline{dl} + \underbrace{\int_{2} \overline{H} \cdot \overline{dl}}_{\overline{H} \perp \overline{dl}} + \underbrace{\int_{3} \overline{H} \cdot \overline{dl}}_{H=0} + \underbrace{\int_{4} \overline{H} \cdot \overline{dl}}_{\overline{H} \perp \overline{dl}} = NI$$



$$\therefore \int_{1} \overline{H} \cdot \overline{dl} = NI$$

$$Hl = NI$$

$$\overline{H} = \frac{NI}{l} \hat{a}_z$$

Step (2): Find B

$$\overline{B} = \mu \overline{H}$$

$$\overline{B} = \frac{\mu NI}{l} \hat{a}_z$$

step ③ Find Φ

$$\Phi = \int \overline{B} \cdot \overline{ds}$$

$$= \int_0^{2\pi}\int_0^{R} \frac{\mu N I}{\ell} * \rho \, d\rho \, d\phi$$

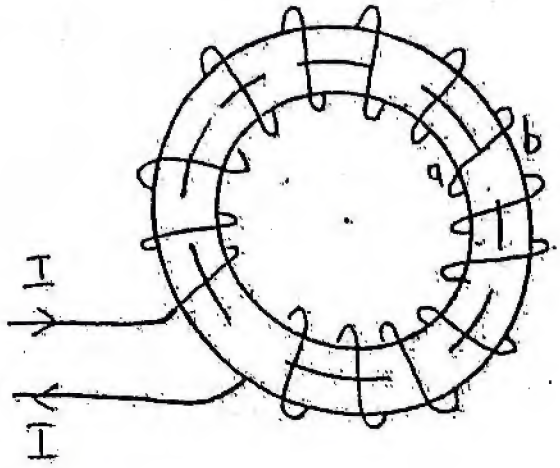
$$= \frac{\mu N I}{\ell} \left[\frac{\rho^2}{2} \Big|_0^R \right] \left[\phi \Big|_0^{2\pi} \right]$$

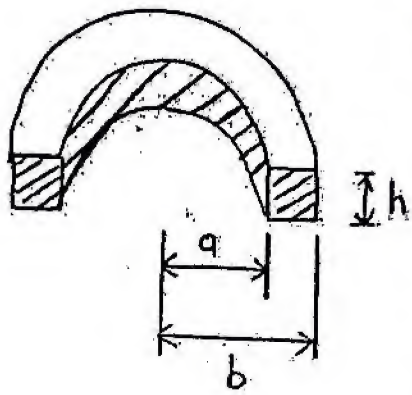
$$\Phi = \frac{\mu N I}{\ell} \pi R^2$$

step ④ : Find Inductance

$$L = \frac{N\Phi}{I} = \frac{\mu N^2}{\ell} \pi R^2$$

$$L = \frac{\mu N^2 \pi R^2}{\ell}$$





Step ① Find H

Using Ampere's law

$$\oint \overline{H} \cdot d\overline{L} = I_{en}$$

$$H * 2\pi r = NI$$

$$\boxed{\overline{H} = \frac{NI}{2\pi r}\,\hat{a}_\phi}$$

Step ② Find B

$$\overline{B} = \mu \overline{H}$$

$$\boxed{\overline{B} = \frac{\mu NI}{2\pi r}\,\hat{a}_\phi}$$

Step ③ Find Φ

$$\Phi = \int \overline{B} \cdot \overline{ds}$$

$$\Phi = \int_0^h \int_a^b \frac{\mu N I}{2\pi r} \, dr \, dZ$$

$$= \frac{\mu N I}{2\pi} \ln r \Big|_a^b * Z \Big|_0^h$$

$$\Phi = \frac{\mu N I h}{2\pi} \ln\left(\frac{b}{a}\right)$$

Step ④ Find inductance

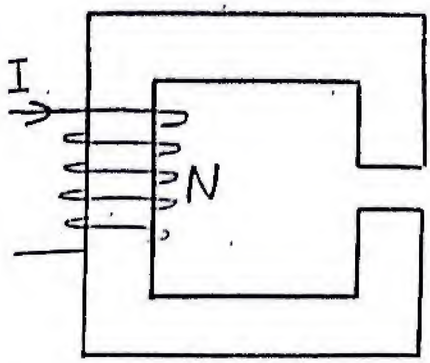
$$L = \frac{N\Phi}{I}$$

$$L = \frac{\mu N^2 h}{2\pi} \ln\left(\frac{b}{a}\right)$$

Magnetic Circuits

Final 2014 / 2015

Q 2-C



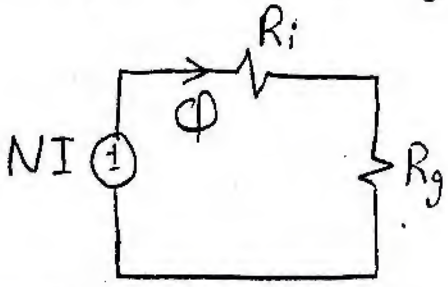
- $A_c = 16 \, cm^2$
- $L_c = 30 \, cm$
- $L_g = 0.6 \, mm$
- $N = 400$
- $I = 1.5$

BH curve of iron is given

Find B_g

Solution

The equivalent Magnetic circuit is



Let $A_i = A_g$

$\therefore B_i = B_g$

$$\boxed{NI = H_i L_i + H_g L_g}$$

$$NI = H_i L_i + H_g L_g$$

$$(400)(1.5) = H_i (0.3 - 0.6 \times 10^{-3}) + H_g (0.6 \times 10^{-3})$$

$$600 = 0.3 H_i + 0.6 \times 10^{-3} \frac{B_g}{\mu_0}$$

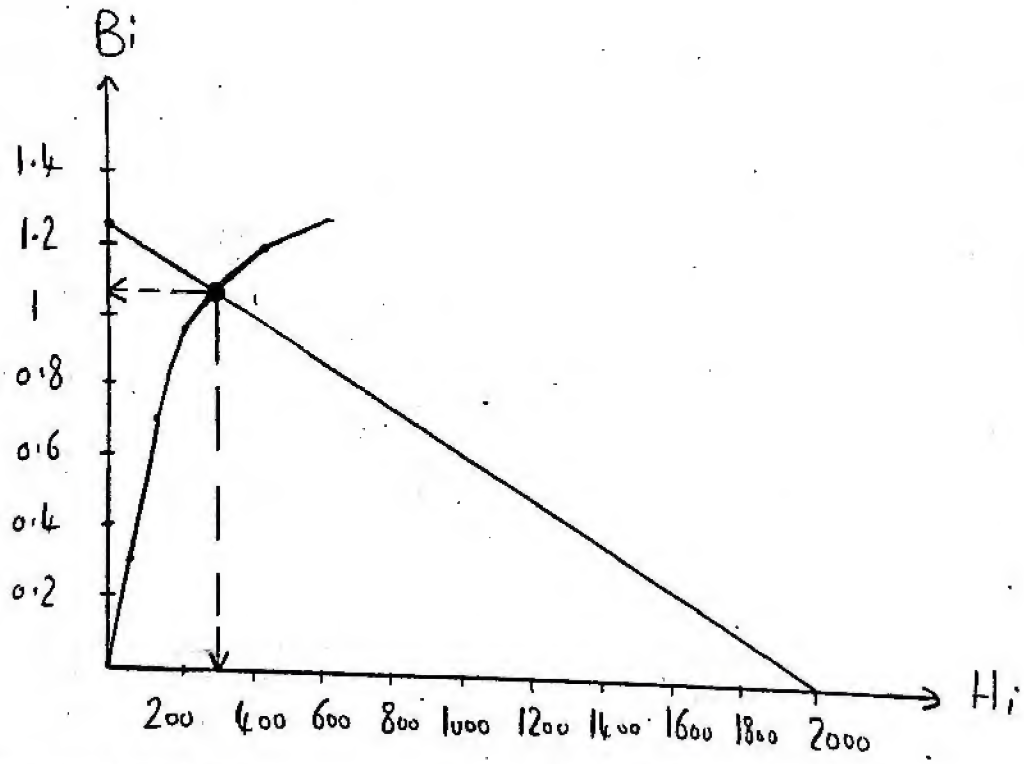
$$\boxed{B_g = B_i}$$

$$\therefore 0.3 H_i + 477.46 B_i = 600$$

$$\boxed{B_i = 1.256 - 6.28 \times 10^{-4} H_i}$$: Draw this relation

at $H_i = 0 \rightarrow B_i = 1.256$

at $B_i = 0 \rightarrow H_i = 2000$



$$\boxed{\therefore B_i \simeq 1.1 T}$$ $$\boxed{H_i = 250 A/m}$$

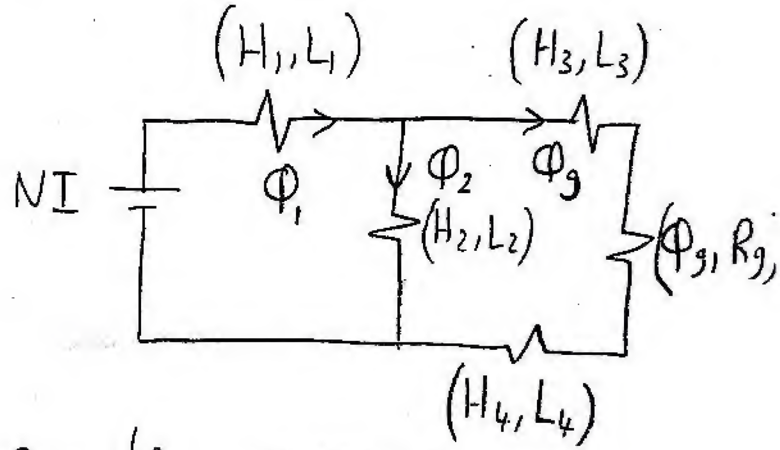
$$\boxed{\therefore B_g = 1.1 T}$$

Q2-c Final 2016

* $L_1 = 40\,cm$, $L_2 = 24\,cm$, $L_3 = L_4 = 26\,cm$, $L_g = 0.025\,cm$
* $A_1 = 40\,cm^2$, $A_2 = 12\,cm^2$, $A_3 = A_4 = 25\,cm^2$, $A_g = 26\,cm^2$
* $N = 500$
* $\Phi_g = 2\,mWb$

Find I, L (inductance)

Solution



→ For right branch

$\Phi_g = 2\,mWb$

$B_g = \dfrac{\Phi_g}{A_g} = 0.77\,T$, $R_g = \dfrac{L_g}{\mu_0 A_g} = 76516.8\,AT/Wb$

$\therefore\ B_3 = B_4 = B_g = 0.77\,T$

From BH curve → $H_3 = H_4 = 120\,AT/m$

→ For center branch

$H_2 L_2 = H_3 L_3 + H_4 L_4 + \Phi_g R_g$

$H_2 (0.24) = 215.4 \implies H_2 = 897.6\,AT/m$

From BH curve $\rightarrow$ $B_2 = 1.32\,T$

$\therefore \Phi_2 = B_2 A_2 = 1.58\,mWb$

For Left Branch

$\rightarrow \Phi_1 = \Phi_2 + \Phi_g = 3.58\,mWb$

$\rightarrow B_1 = \frac{\Phi_1}{A_1} = 0.895\,T$

$\rightarrow$ From BH-curve $\rightarrow$ $H_1 = 150\,AT/m$

$\therefore NI = H_1 L_1 + H_2 L_2$

$NI = 150(0.4) + 897.6(0.24)$

$NI = 275.42$

$N = 500 \Rightarrow \boxed{I = 0.55}$

To get $L \Rightarrow L = \frac{N^2}{R_{eq}}$

$R_{eq} = R_1 + \left[R_2 \,//\, (R_3 + R_4 + R_g)\right]$

$$R_1 = \frac{l_1}{\mu_1 A_1} \quad , \mu_1 = \frac{B_1}{H_1} = 5.96 * 10^{-3}$$

$$\therefore R_1 = 16778.5 \ AT/Wb$$

$$R_2 = \frac{l_2}{\mu_2 A_2} \quad , \mu_2 = \frac{B_2}{\mu_2} = 1.47 * 10^{-3}$$

$$R_2 = 136054.4 \ AT/Wb$$

$$R_3 = R_4 = \frac{l_3}{\mu_3 A_3} \quad , \mu_3 = \frac{B_3}{\mu_3} = 6.416 * 10^{-3}$$

$$R_3 = R_4 = 16209.4 \ AT/Wb$$

$$R_{eq} = R_1 + \left[R_2 \, // \, (R_3 + R_4 + R_g) \right]$$

$$R_{eq} = 77275.5 \ AT/Wb$$

$$L = \frac{N^2}{R_{eq}} \Rightarrow L = 3.2 \ H$$

مجالات كهرومغناطيسية

AIN SHAMS UNIVERSITY
FACULTY OF ENGINEERING
Department of Electrical Power and Machines
2nd Year, Electrical Engineering

1st Semester, 2015/2016	**Course Code: *EPM 211***	***Time Allowed : 3.00 Hrs***
Final Term Exam.	**Electromagnetic Fields**	**Date: 18/01/2016**
Instructor/s: Prof. Dr. Rizk. M. Hamouda	**Assoc. Prof. Dr. Adel A. E. M. S. Ahmed**	**Dr. Yasser Sabry**
The Exam Consists of Four Questions in Five Pages.	***Total Marks: 110 Marks***	**1 / 3**

تعليمات هامة
1) حيازة التليفون المحمول مفتوحا داخل لجنة الأمتحان يعتبر حالة غش تستوجب العقاب وإذا كان ضرورى الدخول بالمحمول فيوضع مغلق فى الحقائب.
2) لا يسمح بدخول سماعة الأذن أو البلوتوث.
3) لايسمح بدخول أي كتب أو ملازم أو أوراق داخل اللجنة والمخالفة تعتبر حالة غش.

General Instructions:

- *Please read the examination paper carefully*
- *The examination consists of 4 Questions in Five Pages.*
- *You should answer All Questions.*
- *Please assume any missing data in a logical manner.*
- *Only a non-programmable calculator may be used.*
- *The Mobile Cell is not allowed to use it in the Exam.*
- *Clearly identify the part of the problem you are solving.*
- *Highlight your answers.*
- *Any intermediate steps you write, will be considered in the marks.*
- *Important constants: $\mu_o=4\pi \times 10^{-7}$ H/m, $\varepsilon_o=8.85 \times 10^{-12}$ F/m.*
- ***Instructions listed above are for guidance and instructor/s should enforce his own perspective.***

Question (1): [25 Mark]

a- **[6 Marks]** In spherical coordinates the following electric fields are $\mathbf{E} = (50/r^2)\ \mathbf{a}_r$ V/m for $0 < r \leq 2$m and $\mathbf{E} = (25/r^2)\mathbf{a}_r$ V/m for $r > 2$m. Find:

1- **(3 Marks)** The potential difference V_{AB} for A(1,0,0)m and B(10,0,0)m.

2- **(3 Marks)** The values of the volume charge densities ρ_v at the points A and B.

b- **[5 Marks]** A thin rod with a uniform charge per unit length ρ_L is bent into the shape of an arc of a circle of radius R. The arc subtends a total angle 2 θ_o, symmetric about the x-axis, as shown in Fig. Find the potential V at the origin.

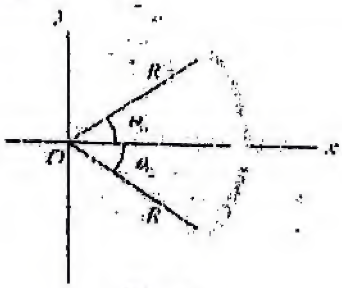


Fig. 1

P. T. O.

c- **[12 Marks]** A charge distribution with a spherical symmetry has density

$$\rho_v = \begin{cases} \rho_o\ , & 0 \le r \le R \\ 0\ , & r > R \end{cases}$$

Determine:

1- **(8 Marks)** The potential 'V' everwhere with spherical symmetry has density. Assuming the reference voltage is at $r = \infty$.

2- **(4 Marks)** The energy stored in region r < R.

d- **[12 Marks]** A capacitor consists of two concentric spherical shells as shown in Fig. 2. The outer radius of the radius of the inner shell is a = 0.1 m and the inner radius of the outer radius of the outer shell is 0.2 m.

1- **(3 Marks)** What is the capacitance C of this capacitor?

2- **(3 Marks)** Suppose the maximum possible electric field at the outer surface of the inner shell before the air starts to ionize is E_{max} (a) = 3 x 10^3 KV/m. What is the maximum possible charge on the inner capacitor?

3- **(3 Marks)** When E (a) = 3 x 10^3 KV/m, what is the potential difference between the shells?

4- **(3 Marks)** What is the maximum amount of energy stored in this capacitor?

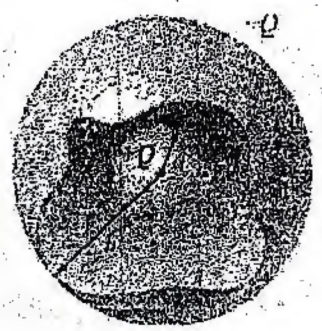


Fig.2

Question (2): [35 Mark]

a- **[10 Marks]** Using Laplace's or Poisson's equation:

1- **(6 Marks)** Determine the voltage as function for a cylindrical capacitor (coaxial cable). The inner conductor is located at ρ = a and the outer conductor is located at ρ = b, where a < b. Charge only exists on the surface of the conductor. Assume that the voltage on the inner conductor is V_o and the outer conductor is grounded. You should draw the figure before you attempt to solve for voltage. Also, you must demonstrate that your answer satisfies either Laplace's or Poisson's equation.

2- **(4 Marks)** Find the capacitance of the coaxial cable.

b- **[10 Marks]** A square wire loop of size a x a lies in the xy plane with its center at the origin and sides parallel to the x and y axes. A counterclockwise current 'I' runs around the loop. Show that the magnetic flux density 'B' at a point (0, 0, z) on the axis (z-axis) of the square wire loop is given by

$$\vec{B}(z) = \frac{2\sqrt{2}\,\mu_o I}{\pi}\hat{z}\frac{1}{(4z^2+a^2)}\frac{a^2}{\sqrt{2z^2+2a^2}}$$

c- **[15 Marks]** It is required to establish a flux of 2 mWb in the air gap of the magnetic structure shown in Fig. 1. The structure is made of silicon sheet steel, and its magnetization curve is as shown in the figure. Assume that the coil has 500 turns, and the structure has the following dimensions:

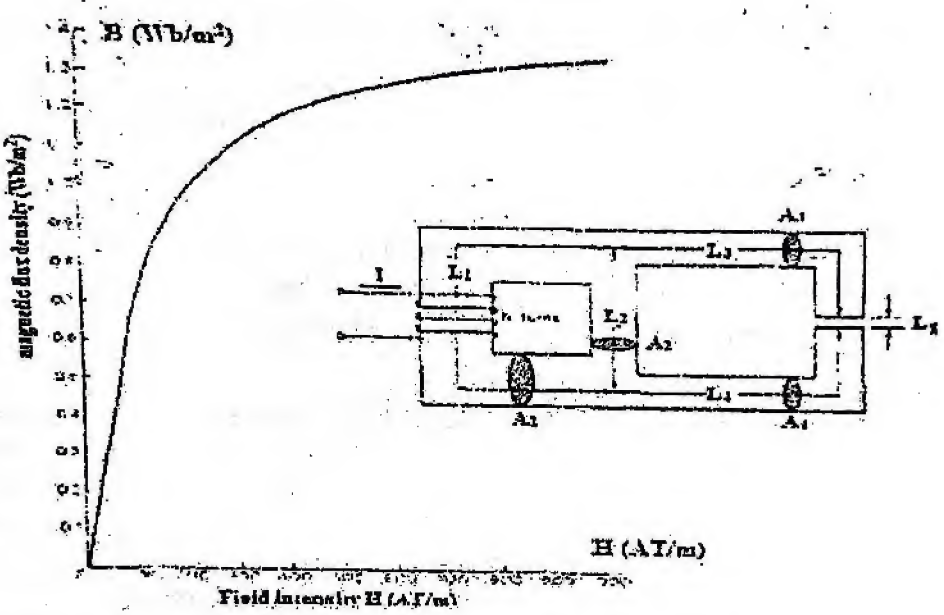
$L_1 = 40$ cm $A_1 = 40$ cm^2 $L_3 = L_4 = 26$ cm $A_3 = A_4 = 25$ cm^2

$L_2 = 24$ cm $A_2 = 12$ cm^2 $L_g = 0.025$ cm $A_g = 26$ cm^2

Find:

1- **(12 Marks)** The current, I in Amp.

2- **(3 Marks)** The self inductance of the coil, L in Henry.



Question (3): [40 Mark]

Instrutions of this questions:

Arrange all your answers in tables as givenbelow each question.Copysimilar tables in your answer sheet and put all you answer inside the tables. Any answer outside the tables will not be considered

1) Mention 4 reasons why the engineers are interested in high frequency signals. Make your answer short and concise.[8 marks].

Answer

1	
2	
3	
4	

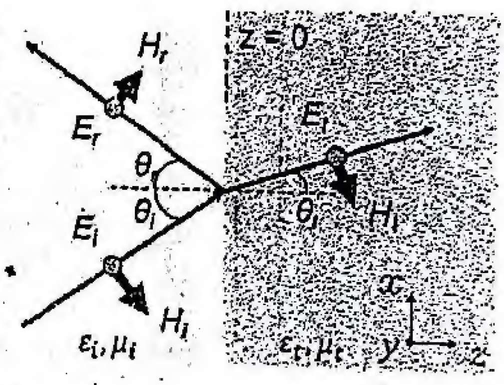
II) Indicate whether the following statements are true (T) or false (F) and give the corrected statement, if the provided statement is false **[12 marks]**.

1. The circuit theory is applied when the wavelength λ <<component dimensions
2. Faraday's law indicates that all electric flux begins and terminates on charge
3. The wave front is the locus of points characterized by position of the same phase
4. The attenuation constant and the phase constant are approximately equal in good conductors
5. A single conductor with large cross section area is preferred for the power transmission lines
6. Microwave ovens operateat 2.45 GHz at which water has maximum absorption of EM wave

Answer

No.	T or F	Brief justification, if the provided statement is false السبب باختصار إذا كانت العبارة خطأ
1		
2		
3		
4		
5		
6		

III) Assume aplane wave propagating with a k_x and k_z components in the z<0 region and with the electric field E_i and magnetic field H_i vectors in the directions shown in the figure. The plan wave is incident on the interface z=0 and accordingly there will be reflected wave and transmitted wave. Answer the following questions**[10 marks]**:



1. Write down the general form of the equations of the electric field vectors E_i, E_r and E_t assuming the reflection coefficient is r and the transmission coefficient is t **(3 marks)**
2. Apply the phase matching condition and find the relation between θ_i and θ_t **(3 marks)**
3. Apply the tangential boundary condition and find the two equations connecting r and t to the medium wave impedance and the angles **(4 marks)**

Answer

No. 1 Eqn (1)	$\mathbf{E}_i =$
No. 1 Eqn (2)	$\mathbf{E}_r =$
No. 1 Eqn (3)	$\mathbf{E}_t =$
No.2 Condition (1)	
No. 2 Condition (2)	
No. 2 Eqn (1)	
No.3 Condition (1)	
No.3 Eqn (1)	
No.3 Condition (2)	
No.3 Eqn (2)	

IV) Assume a uniform optical plane wave with a frequency 474 THz (Note that 1 THz $=10^{12}$ Hz) is emitted from a laser source and propagating in free-space (ε_o= 8.85 x 10^{-12} F/m, μ_o=4π x 10^{-7} H/m) in the positive x-direction. The time average power density S_{avg} = 10^3 W/m^2. Determine the following including the appropriate units **[10 marks]**:

1. The wavelength λ **(4marks)**
2. The angular wavenumber$k$ **(2 marks)**
3. The peak value of the electric field **(4marks)**

Answer

No.	Expression	Substitution	Final answer
1	λ=	λ=	λ=
2	k=	k=	k=
3	E=	E=	E=

END of Examination

Exam. Date : 18th Jan. 2016

Course Examination Committee
Prof. Dr. Rizk. M. Hamouda
Assoc. Prof. Dr. Adel A. E. M. S. Ahmed
Dr. Yasser Sabry

N.B.:

Vacuum permittivity. ε_0= 8.854*10^{-12} F/m

$dv = \rho d\rho d\varphi dz$ (cylindrical), $dv = r^2 sin\theta dr d\theta d\varphi$ (spherical)

$$\nabla.\bar{A} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial\varphi} + \frac{\partial A_z}{\partial z} \quad (cylindrical),$$

$$\nabla.\bar{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r sin\theta}\frac{\partial(A_\theta sin\theta)}{\partial\theta} + \frac{1}{r sin\theta}\frac{\partial A_\varphi}{\partial\varphi} \quad (spherical)$$

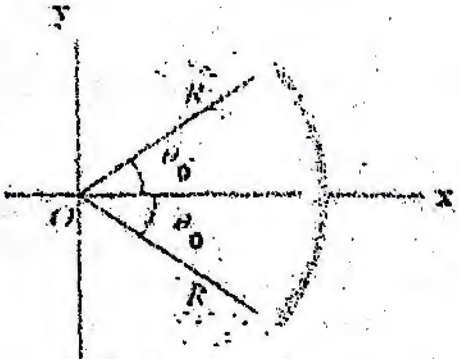
$$\nabla V = \frac{\partial V}{\partial\rho}a_\rho + \frac{1}{\rho}\frac{\partial V}{r\partial\varphi}a_\varphi + \frac{\partial V}{\partial z}a_z \quad (cylindrical)$$

$$\nabla V = \frac{\partial V}{\partial r}a_r + \frac{1}{r}\frac{\partial V}{\partial\theta}a_\theta + \frac{1}{r sin\theta}\frac{\partial V}{\partial\varphi}a_\varphi \quad (spherical)$$

$$\boxed{\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \quad \text{(rectangular)}}$$

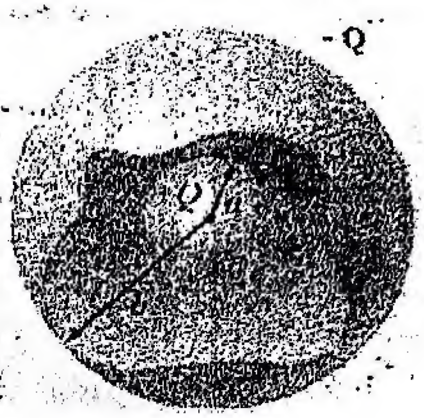
$$\boxed{\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) + \frac{1}{\rho^2}\left(\frac{\partial^2 V}{\partial\phi^2}\right) + \frac{\partial^2 V}{\partial z^2} \quad \text{(cylindrical)}}$$

$$\boxed{\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2} \quad \text{(spherical)}}$$



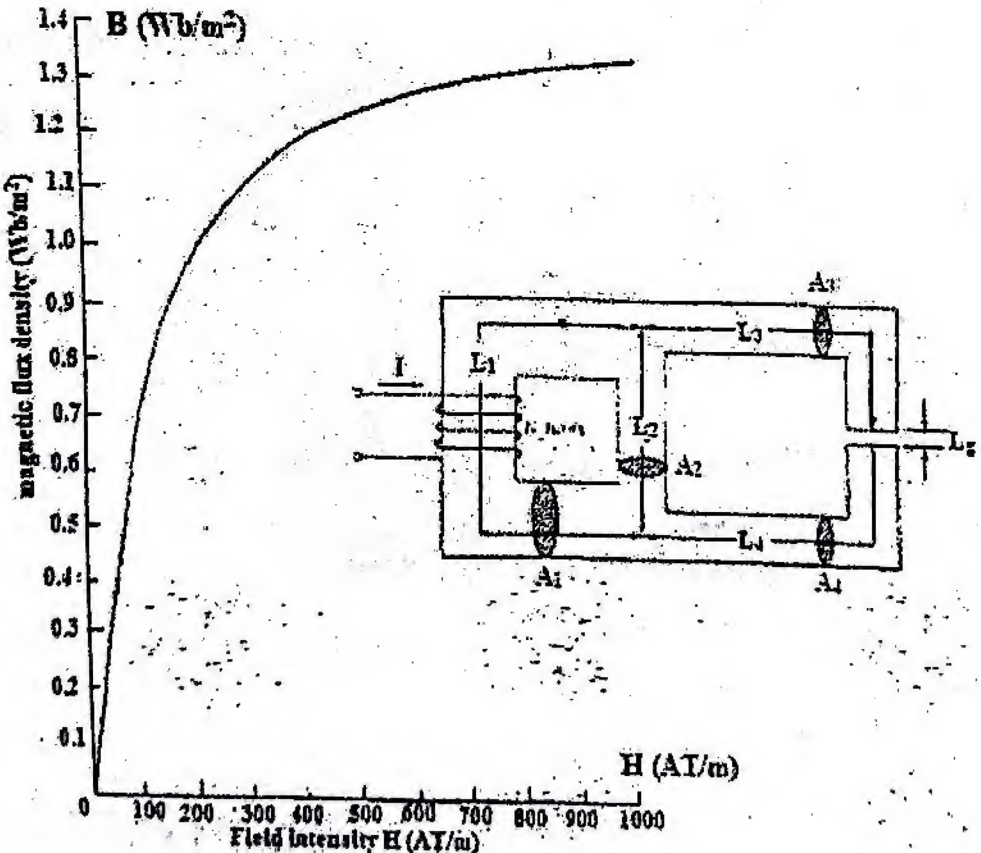
Y
R
θ_0
O
θ_0
R
X

Fig. 1



Q

Fig.2



B (Wb/m²)
magnetic flux density (Wb/m²)
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
100 200 300 400 500 600 700 800 900 1000
H (AT/m)
Field intensity H (AT/m)
I
L_1
L_2
L_3
L_4
L_g
A_1
A_2
A_3
A_4

AIN SHAMS UNIVERSITY
FACULTY OF ENGINEERING

Department of Electrical Power and Machines
2^{nd} Year, Electrical Engineering

1st Semester Academic Year 2014-2015 **Exam Date: 05.12.2014** **Exam Time: 2 Hours**

Electromagnetic Fields EPM 211

The Exam consists of TWO Questions in TWO page **Total Marks: 30 Marks** **1/2**

Answer All Questions

Question 1: (15 Marks)

a) A circular ring has a radius a = 2m lies in z = 0 plane with its center at the origin. If ρ_L= 10 nc/m. Find:

1- A point charge at the origin which could produce the same electric field at P(0, 0, 5). **{3 Marks}**

2- The potential difference between the points $P_1(0, 0, 10)$ and $P_2(0, 0, -10)$. (Use infinity as a potential reference, i.e., ground) **{3 Marks}**

b) An infinitely long metallic cylinder with radius 0.1 m is evenly charged with a charge 1 c/m (charge per meter of length)

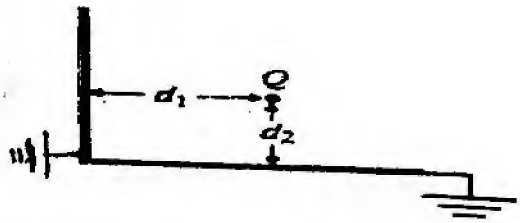
1- Calculate the electric field outside the cylinder.

2- Calculate the electrostatic potential outside the cylinder. (Use r = 50 m as a potential reference) **{2 Marks}**

3- How much energy do you need to move an electron (charge = -1.6x 10-19 c) from r = 1m to r = 2m? **{2 Marks}**

{1 Mark}

c) A positive point charge Q is located at distance d1 and d2, respectively, from two grounded perpendicular conducting half-planes (infinitely long), as shown in the figure. Determine the forces on Q caused by the charges induced on the planes. **{4 Marks}**

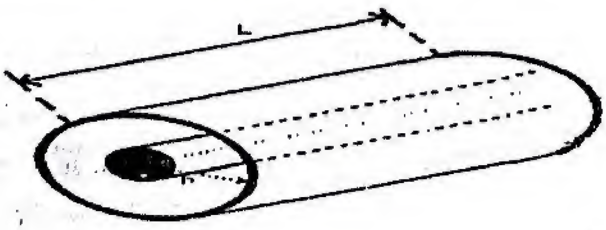


Question 2: (15 Marks)

a) To solve an electro-magnetic problem most engineers rely on numerical solutions to the Poisson or Laplace equations. However, for simple geometries we can solve the Laplace equation analytically. Write down the Laplace equation for a cylinder capacitor (vacuum between the cylinders, i.e., zero charge density) where the cylinders have radii **a** and **b**. The length of the capacitor is much longer than the radii (L>>b).

1- Solve the Laplace equation for the potential between the cylinders with the boundary conditions that the voltage is zero at the inner cylinder (Dirichlet boundary condition) and the electric field is 1 V/m (outwards) just inside the outer cylinder (Von Neumann boundary condition). **{3 Marks}**

2- What is the charge on the inner cylinder (per length of the cylinder)? **{2 Marks}**



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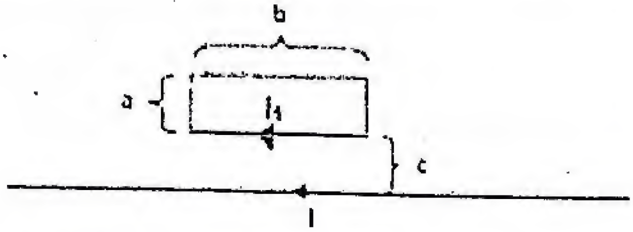
b) Consider a long thin wire carrying a dc current: I = 2 A and the square loop is carrying a dc current I1 as shown in the figure below. Assume that a = 4 cm, b = 7 cm and c = 4 cm. Calculate:

1- The magnetic flux is crossing the square loop when the current $I_1 = 0$. {2 Marks}

2- The total force on the square loop when the current $I_1 = 5$ A. {3 Marks}



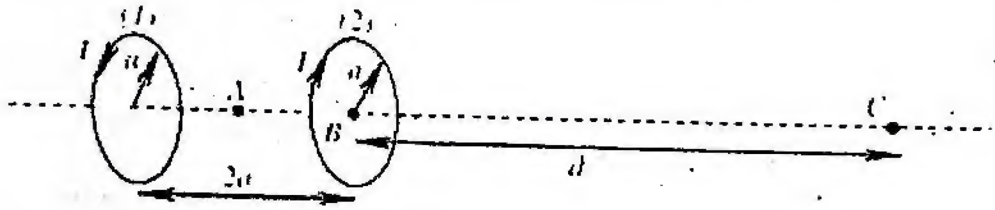
c) Two loops, made of thin wire carry equal and opposite currents as shown the figure below. The radius of each loops is a and the distance between the loops is 2a. The loops are in free space and placed parallel to each other, so that they are both perpendicular to the axis A-C as shown (that is, like two wheels on the axle). Calculate:

1- The magnetic flux density at point A (midway between the two loops). {2 Marks}

2- The magnetic flux density at point B (at the center of loop No. 2). {2 Marks}

3- The magnetic flux density at point C if d >> a. {1 Mark}

Note: In all three cases give both direction and magnitude of the magnetic flux density.



All The Best

Prof. Dr. Rizk M. Hamouda **Assoc. Prof. Dr. Adel A. M. S. Ahmed**

N.B.:

Vacuum permittivity ε_o= 8.854*10-12 F/m

$dv = r\,dr\,d\varphi\,dz$ **(cylindrical)**, $dv = r^2 \sin\theta\, dr\, d\theta\, d\varphi$ **(spherical)**

$$\nabla\cdot\vec{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z} \quad \text{(cylindrical)},$$

$$\nabla\cdot\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial(A_\theta \sin\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial\varphi} \quad \text{(spherical)}$$

$$\nabla V = \frac{\partial V}{\partial r}a_r + \frac{1}{r}\frac{\partial V}{\partial\varphi}a_\varphi + \frac{\partial V}{\partial z}a_z \quad \text{(cylindrical)}$$

$$\nabla V = \frac{\partial V}{\partial r}a_r + \frac{1}{r}\frac{\partial V}{\partial\theta}a_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial\varphi}a_\varphi \quad \text{(spherical)}$$

Laplace Equation:

Cartesian Coordinates	$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0.$
Cylindrical Coordinates	$\Delta f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2} = 0$
Spherical Coordinates	$\Delta f = \frac{1}{\rho^2}\frac{\partial}{\partial\rho}\left(\rho^2\frac{\partial f}{\partial\rho}\right) + \frac{1}{\rho^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{\rho^2\sin^2\theta}\frac{\partial^2 f}{\partial\varphi^2} = 0.$

AIN SHAMS UNIVERSITY
FACULTY OF ENGINEERING
Department of Electrical Power and Machines
2[nd] Year, Electrical Engineering

1st Semester Academic Year 2014-2015 | Exam Date: 12.01.2015 | Exam Time: 3 Hrs

Electromagnetic Fields EPM 211

The Exam consists of THREE Questions in THREE pages | Total Marks: 110 Marks | 1/3

Answer All Questions

Instructions:

1) *Clearly identify the part of the problem you are solving.*
2) *Highlight your answers.*
3) *Do not miss units or vector directions, each has weight of 0.25 mark.*
4) *Any intermediate steps you write, are considered in the marks.*
5) ***Important constants:*** $\mu_0=4\pi \times 10^{-7}$ *H/m,* $\varepsilon_0=8.85 \times 10^{-12}$ *F/m,* $c_0=3 \times 10^8$ *m/s*

Question 1 (33 Marks)

a- The annular surface 1 cm<ρ<3cm, $Z = 0$, carries the nonuniform surface charge density $\rho_s = 5\rho$ nC/m^2.

i. Find V at $P(0, 0, 2\text{ cm})$ if $V = 0$ at infinity. (6 Marks)

ii. From the result of part (i), find the electric field at point P. (4 Marks)

b- Consider a uniform charged dielectric sphere of radius a = 0.5 m and $\rho_v = 2$ mc/m^3.

i. Find the electric field intensity **E** at r = 0.2 m, r = 0.5 m and r = 2 m. (4 Marks)

ii. Plot the variation of the electric field intensity versus r. (2 Marks)

iii. Find the electrostatic potential V at r = 0.2 m, r = 0.5 m and r = 2 m. (V at infinity=0) (4 Marks)

iv. Plot the variation of the electrostatic potential V versus r. (2 Marks)

c- A potential field in free space is expressed as $V = 20/(xyz)$V. Find the total energy stored within the cube $1 < x, y, z < 2$ (4 Marks)

d- Two point charges of $-100\pi\mu$C are located at (2, −1, 0) and (2, 1, 0). The surface $x = 0$ is a grounded conducting plane. (*a*) Determine the surface charge density at the origin. (*b*) Determine ρ_s at $P(0, h, 0)$. (7 Marks)

Question 2 (32 Marks)

a) Determine the potential at the free nodes in the potential system of Fig. 1 using the iteration method. Assume that the per unit residues should be ≤ 0.02 (10 Marks)

(Hint: 2 iterations (attempts) are enough)

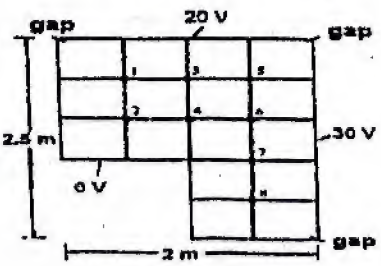


Fig. 1

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b) Find the magnetic field density **B** at point P due to the stationary current 5A going through the following wire configuration (Fig. 2). Assume that the current loop wire is located in x-y plane. (10 Marks)

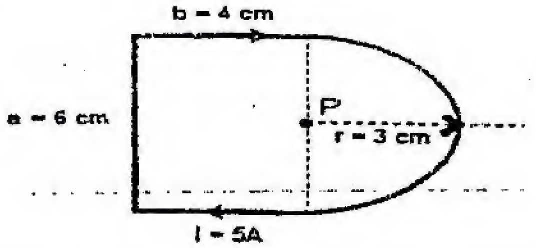

Fig. 2

c) The magnetic circuit shown in Fig. 3 has the following dimensions: $A_c = 16\ cm^2$, $l_c = 30$ cm, $l_g = 0.6$ mm $N = 400$ turns and $I = 1.5$ A. The core is made of a material with ***B-H*** relationship given below in the table. Determine the magnetic field density in the gap B_g. (12 Marks)

B (Tesla)	0	0.3	0.7	0.98	1.13	1.2	1.27	1.3	1.33	1.35
H (A.T/m)	0	50	100	200	300	400	600	700	800	1000

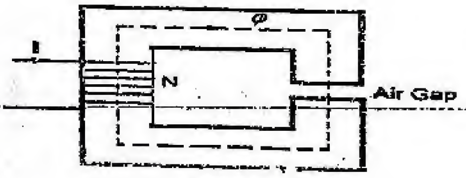

Fig.3

Question 3 (45 Marks)

Question 3A (20 Marks)

i. Write the general differential form equations of Maxwell in the time domain. (4 Marks)

ii. Write the differential form of Maxwell equations in the time domain for lossless, source free medium in terms of electric field intensity vector **E** and magnetic field intensity vector **H**. (4 Marks)

iii. Using Maxwell equations obtained in (b), Drive the wave equation in terms of time and space for lossless, source free medium, (Assume only space variations in z-direction). (4 Marks)

iv. Use the field time dependence function, $e^{j\omega t}$, to reduce the number of variables in the wave equation obtained in (c) and re-write the wave equation in terms of space only. From the derivation, write the propagation constant term for such medium. (4 Marks)

v. Use the field time dependence function, $e^{j\omega t}$, to re-write the differential form of Maxwell equations obtained in (a) in the frequency domain in terms of electric field intensity vector **E** and magnetic field intensity vector **H**. (4 Marks)

Hint: $\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$

P.T.O